Math 14 syllabus

Math 14, 55130, Spring 2010, MWF 9:15 - 10:20, O’Connor 104.
Professor Schaefer, O’Connor 309, 554-6899, eschaef@scu.edu,
website: google “Ed Schaefer”

Office Hours: Tuesday 1 - 2:10, Wednesday 8:30 - 9, Thursday 4 - 5.


In this course, we will study functions of several variables.

There will be several in-class quizzes, I will drop the lowest score. If you warn me before the day of the quiz, you can take it early.

Grade: Homework 15%, Quizzes 5%, Midterm 30% Final 50%.

I will drop the lowest two lecture's worth of homework scores, but I EXPECT YOU TO DO ALL OF THE HOMEWORK ASSIGNMENTS. No late homework is accepted. Please turn in homework stapled and folded (so it is 4 1/4” x 11”) so that your name is on top and visible. Show your work on homework, quizzes and exams. Show enough work to demonstrate to the grader that you did your own work and understand how to solve the problem.

GETTING HELP: There is a lot of help available on campus. The earliest available tutoring outside of my office hours is private tutoring offered by the Drahmann Center (DC) in Benson Center 214; the phone number is 4318. If you need a private tutor, get one early. In a few weeks, the DC will organize drop-in tutoring, 3 evenings a week. I will announce the schedule and place when I find out. The DC can also help with test anxiety and study skills.

There will soon be drop-in tutoring in O’Connor 31 (Sussman room) available during several of the hours Monday-Friday 8-5. I will announce the schedule when I receive it. The tutors are honors math majors and are usually excellent. Go into the room and ask who the tutor is. They will help you when they can. If you are really stressed out, there is the counseling center at the Benson Center 201, phone number 4172. You can visit or phone to make an appointment to speak with a counselor.

In this course, students will learn to:

- Solve problems, including choosing and developing appropriate methods, as well as communicating mathematical ideas effectively. In this course we will emphasize the use of integrals and multivariable calculus as an important problem-solving tool.

- Use mathematical reasoning and deduction to draw valid conclusions from given information. For example, we will learn to analyze surfaces and their volumes by studying properties of their antiderivatives.

- Use and understand mathematical ideas from multiple and interconnected perspectives, including algebraic, geometric, analytical and numerical points of view. We will combine geometric visualization with careful analytical reasoning to solve problems and connect our ideas to other disciplines.
• Understand significant mathematical ideas and results in addition to mastering efficient computational techniques. Beyond computational proficiency, we will strive to understand the meaning of our results, as well as encountering some central theorems of mathematics.

In addition to providing you with a good foundation in a fundamental area of mathematics, this course will also contribute to your skills and logical perspective that will be applicable to many other courses requiring mathematical methods and careful reasoning.

Academic Integrity: The penalty for cheating is a failing grade for the course, and the University may take further disciplinary action. All of the work that you turn in should be your own, and not that of a classmate or copied from another source. See http://www.scu.edu/studentlife/resources/academicintegrity/index.cfm for more information.

Disability accommodation policy: To request academic accommodations for a disability, students must contact the Disability Resources Office located in Benson room 216, (408) 554-4111; TTY (408) 554-5445. Students must provide documentation of a disability to Disability Resources prior to receiving accommodations.

Homework

Due Wed 3/31. Read the syllabus up to this point.

Due Wed 4/7: [10.1] 2a-€, 3, 4a, 7-13, 18-21, 23, 35, 40, 44. (Sol’ns in back to 11 and 13 are switched). [15.3] 1-10, 12, 13, 15, 19-22, 29, 30.
Math 14 Spring 2010 homework

Due Fri 4/9: [16.1] 3, 8, 9, 19-21 (hint on 20: square both sides), 28. A. Sketch the graph of \( h(x, y) = 3 - x^2 - y^2 \). B. Level curves for the function \( f(x, y) \) are shown below. Estimate \( f(-1, 1) \) and \( f(1, 1) \). From outside in are \( f = 10, f = 20, f = 30, f = 40 \). Describe the graph of \( z = f(x, y) \). C. Draw level curves for the function \( g(x, y) = x^2 - y^2 \) for \( g = -2, -1, 0, 1 \) and \( 2 \). Describe what kind of graph this function has. D. Describe the level surfaces of \( j(x, y, z) = x + y + z \).

Due Wed 4/14: For problems [16.3] 7-12, find \( \partial f / \partial x, \partial f / \partial y, \partial^2 f / \partial x^2, \partial^2 f / \partial y^2, \partial^2 f / \partial x \partial y \). For [16.3] 9, 12 also compute \( \partial^2 f / \partial y \partial x \) to confirm it’s the same as \( \partial^2 f / \partial x \partial y \). [16.3] 21, 27, 28. E. Wind-chill index \( I \) is a function of actual temperature \( t \) in centigrade and wind speed \( v \) in km/hour. So \( I = f(t, v) \). The chart below gives values of \( I \) for some pairs \((t, v)\). The numbers across the top \((10, 20, 30, 40, 50)\) are \( v \)’s, the numbers down the left \((20, 16, 12, 8)\) are \( t \)’s. The numbers inside are \( I \)’s. So \( f(12, 20) = 5 \). Estimate \( f_t(12, 20) \) and \( f_v(12, 20) \) (there are several acceptable answers). What are the practical interpretations of each? I’m looking for sentences like “If the velocity is 20 and the temperature increases from 12 then the windchill . . .”

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F. The pressure in the first octant is \( P(x, y, z) = x^2 + yz \). Use partial derivatives to estimate the pressure change which results from moving from \((1, 2, 3)\) to i) \((1.1, 2, 3)\) and ii) \((1, 2.2, 3)\). G. Level curves for the function \( f(x, y) \) are given below. Use them to estimate \( f_x(1, 1) \) and \( f_y(1, 1) \).

H. i) Find the linearization to \( f(x, y) = x^3 + y^2 \) at \((2, 1)\). ii) Use it to estimate \( f(2.2, .9) \). iii) Find the actual \( f(2.2, .9) \) on your calculator to see how good the estimate was. iv) Find the equation of the tangent plane to \( z = x^3 + y^2 \) at \((2, 1, 8)\).

I. Do the same as in H. with \( f(x, y) = \sqrt{1 + x + 6y} \) (i.e. use \((2, 1)\) again etc). J. i) Use a linearization to approximate \( \sin(0.01) + \cos(0.02) + \tan(-0.03) \). ii) Find the actual value on your calculator for comparison. K. The volume of a cone is \( \frac{\pi}{3} rh^2 \). You intend to build a cone with \( r = 3 \text{cm} \) and \( h = 5 \text{cm} \) but both measurements may be off by as much as \( 0.1 \text{cm} \). Use the principle of the differential to approximate the maximal error you might get in volume. Use your calculator to give a decimal answer. L. You intend to build a box with height \( 50 \text{cm} \), width \( 60 \text{cm} \) and depth \( 70 \text{cm} \). Each measurement may be off by as much as \( 0.1 \text{cm} \). Use the principle of the differential to approximate the maximal error you might get in volume. Does the answer surprise you? Calculate \((50.1)(60.1)(70.1) - (50)(60)(70)\) for comparison.

M. Given \( f(1, 2) = 3, f(2, 2) = 3.5 \) and \( f(1, 3) = 2.8 \), estimate \( f(1.5, 2.5) \) and \( f(2.5, 1.5) \).
Due Fri 4/16: [16.4] 1, 2, 17, 18, 20, 28 (for 28, evaluate each side and check they’re the same). N. \( z = f(x, y) \), \( x = r \cos(\theta) \), \( y = r \sin(\theta) \). Write \( \partial z / \partial r \) and \( \partial z / \partial \theta \) in terms of \( r, \theta, \partial f / \partial x \) and \( \partial f / \partial y \). Problem O. \( w = f(x, y, z) \), \( x = r \sin(\phi) \cos(\theta) \), \( y = r \sin(\phi) \sin(\theta) \), \( z = r \cos(\phi) \). Write \( \partial w / \partial \theta \) in terms of \( \partial f / \partial x, \partial f / \partial y, \partial f / \partial z, \rho, \phi \) and \( \theta \).

Due Wed 4/21: [16.6] 1, 2, 4, 9, 12, 19, 20, 27, 32, 34. [16.7]: 1a, 9a, 14a, 21, 24, 36. P. A contour map is shown below of barometric pressure (in millibars) during a hurricane. We can consider these to be level curves for the pressure function. i) Estimate the directional derivative of the pressure function at Cancun in the direction of the eye of the hurricane. ii) What are the units of the directional derivative (what per what?). Q. Estimate \( \nabla f(3, 4) \) for the function whose level curves are shown below. Think about length and direction. Answer in the form \( ai + bj \). R. If \( f(x, y) = x^2 + 4y^2 \), find the gradient vector \( \nabla f(2, 1) \) and use it to find the tangent line to the level curve \( f(x, y) = 8 \) at \((2, 1)\). Sketch the level curve, tangent line and gradient vector.

Due Fri 4/23: [17.1] 1, 6, 25, 28 (in section 17.1, they are looking for local maxima and minima). S. Find the maximal and minimal values of \( f(x, y) = x^2 + y^2 - 6y \) over the triangular region bounded by \( y = x \), \( y = -2x \) and \( y = 4 \).

Due Wed 4/28: [17.3] 1, 6, 10, 21.

Due Wed 5/12: [18.1] 1, 2, 10, 11, 13, 17, 22, 37, 39, 40. T. You assign \( xy \)-coordinates to the inside face of a vertical dam. It’s shape is a trapezoid (so all four sides are line segments). The corners are at \((0, 0)\), \((10, -10)\), \((20, -10)\), \((30, 0)\) (distances are measured in meters). Pressure at \((x, y)\) is given by \( \delta g D \) where \( \delta = 1000 \) (kilograms per meter\(^3\)) is the density of the water, \( g = 9.8 \) (meters per second\(^2\)) is the magnitude of gravity and \( D \) is the depth (in meters) of the water at \((x, y)\). So the units of pressure measure \( \text{force per area} \). Compute the total force from pressure against the dam. (Aside: We see the units of \( \delta g D \) are \( \text{kilograms per meter}^2 \text{ per second}^2 \) which measures mass acceleration \( \frac{1}{\text{area}} = \frac{\text{force}}{\text{area}} \). Note the units of Newtons are \( \frac{\text{kilograms per meter}^2}{\text{per second}^2} \). So your answer measures Newtons.)

[18.2] 1 \((a > 0)\), 4, 8-11, 17, 25, 29 (just center of mass). You are welcome to use the integral tables at the front and back of your book for things like \( \int \sin^2(x) \). Also, feel free to use symmetry and/or high school geometry to reduce the number of double integrals.
Due Fri 5/14: [18.3] 1, 3, 9 (that square root in the upper limit is not over the x), 14 (OK to use integral tables).

Due Wed 5/19: [18.4] 1, 3, 6, 19, 24, 25, 30 (Ignore the word cylinder. The book uses the word cylinder to describe a surface given by an equation missing a variable.)

[18.5] 4a (just centroid, answer in back: $\bar{x} = \frac{1}{4}$), 5 (just center of mass), 7 (use common sense for $\bar{x}, \bar{y}$, polar helps).

Due Fri 5/21: [18.6] 1, 7, 21, 25, 29, 45 ($a = 1$), 53 (let $a = 1$; use common sense for two of them). U. Find the average value of the function $z^2$ over $x^2 + y^2 + z^2 \leq 9$. Recall the average value of $f$ is $\int_1^1 \int_1^1 f(x, y, z) \, dx \, dy \, dz$, or $\int_1^1 \int_1^1 \int_1^1 f(x, y, z) \, dx \, dy \, dz$, depending on the region of integration.

Due Wed 5/26: [19.1] 1 - 9, 13, 15, 21, 29 (just center of mass, let $a = 1$). V. a) You have a fence coming out of the xy-plane. The base of the fence is the curve $y = x^2$ from (0, 0) to (2, 4). The height of the fence at the point $(x, y)$ is $x$. You break the curve into four pieces of equal length $\Delta s$. Let $P_1 = (x_1, y_1), P_2 = (x_2, y_2), P_3 = (x_3, y_3), P_4 = (x_4, y_4)$ be the midpoints of each of the four pieces. Approximate the area of the fence using symbols from +, $\Delta s, x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ (you may not need all of those symbols). b) If we let the number of pieces of curve go to infinity, then the limit of this process gives you the exact area of the fence (OK, of one side of the fence if you are picky). Find it.

[19.2] 1, 2, 11, 19, 21, 24. On 19, 21, 24, let $a = 1$. On 24, do two problems, first with $F_1$, then again with $F_2$.

Due Fri 5/28: [19.3] 4 (let $a = 1$), 7, 11, 13, 14.


Due Fri 6/4: [19.7] 11, 12, 14, 19.

Some answers in the back: B. Upside down paraboloid. D. The planes normal to $i + j + k$. E. $f_1(12, 20) \approx 1.375$. H. $12x + 16y = 32, 8.800, 8.625$. J. .98000, .97979. L. 1070 cm$^3$. N. $\frac{\partial z}{\partial r} = \frac{\partial}{\partial x} \cos(\theta) + \frac{\partial}{\partial y} \sin(\theta)$. S. Max value 8. T. $8.17 \cdot 10^8$ Newtons (which is about 918 tons).