Solution to Quiz 3. For points in $x^2 + y^2 \leq 4$ (in the $xy$-plane), find the average distance to the origin.

The disk is centered at the origin and has radius 2. The distance from $(x, y)$ to $(0, 0)$ is given by $\sqrt{x^2 + y^2} = r$. The average value of $r$ over the disk is given by

$$\frac{\int \int r \, dA}{\int \int 1 \, dA} = \frac{\int_{0}^{2\pi} \int_{0}^{2} r \, r \, dr \, d\theta}{\text{Area(\bullet)}}$$

$$= \frac{1}{4\pi} \int_{0}^{2\pi} \frac{r^3}{3} \bigg|_{0}^{2} \, d\theta = \frac{1}{4\pi} \int_{0}^{2\pi} \frac{8}{3} \, d\theta = \frac{1}{4\pi} (2\pi)(\frac{8}{3}) = \frac{4}{3}$$