1. (15) On the grid provided above, sketch a graph of one function \( y = f(x) \) with the following properties.

   a) The domain of \( f(x) \) is \([-5, 5]\)
   b) The range of \( f(x) \) is \([0, \infty)\)
   c) \( \lim_{x \to 2} f(x) = 4 \)
   d) \( f(2) < 0 \)
   e) The tangent line to the graph at \( x = 4 \) is \( y - 3 = -(x - 4) \)
   f) \( \lim_{x \to -3^+} f(x) = 2 \)
   g) \( \lim_{x \to -3^-} f(x) = 5 \)

2. (10) The graph of \( y = g(x) \) is provided above.

   a) Indicate on the \( x \)-axis a value \( c \) where the average rate of change on the interval \([3, c]\) is approximately \( \frac{1}{2} \).
   b) For which values of \( a \) is the function \( h(x) = \frac{g(x)}{(g(x))^2 - 4} \) discontinuous at \( x = a \)?
   c) Does the limit

   \[
   \lim_{x \to 2} \frac{1}{2 - \sqrt{g(x)}}
   \]

   exist? Explain.
3. (25) Let

\[ f(x) = \sqrt{2x + 1}. \]

a) Find, by definition, \( f'(a) \).

b) Find the equation of the tangent line to the graph of \( y = f(x) \) at the point \( x = 4 \).

4. (15) Determine the following limits

\[
\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} \quad \lim_{x \to 4} \frac{x^3 - 8}{x - 4} \quad \lim_{x \to 3} \frac{x^3 - 8}{x - 4}
\]

5. (10) Show that \( x^5 - 4x = -2 \) has a solution for some number \( x \) in the interval \((0,1)\). Explain your reasoning briefly, but thoroughly.

6. a) (5) What is the definition of \( f(x) \) is continuous at \( x = a \)?

b) (10) At which points is the following function discontinuous? Explain your answer thoroughly.

\[
f(x) = \begin{cases} 
  x^4 - 9x, & x < 2 \\
  2x^2 - 10, & 2 \leq x < 5 \\
  \frac{16x}{7-x}, & x > 5 
\end{cases}
\]