

Name: \_\_\_\_\_

Show all work—to me, not your neighbor.

1. Evaluate

a)  $\int (2x^4 - \frac{6}{x^3} + \pi^3) dx.$

b)  $\int \tan^2 \theta d\theta.$  (Hint:  $\tan^2 \theta = \sec^2 \theta - 1.$ )

2. The table below shows the values for the velocity  $v(t)$  of a particle traveling on the number line for some values of  $t$ .

$t$	0.0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$v(t)$	2.1	1.3	-0.4	-0.7	-0.2	0.8	1.5	2.6	3.0

Using the Trapezoid rule and Simpson's rule with  $n = 8$ , approximate the displacement of the particle over the first two seconds, given by the integral

$$\int_0^2 v(t) dt$$

3. Evaluate the following integrals

a)  $\int x^2 \cos(2x^3) dx$

b)  $\int_2^{12} \sqrt{4x+1} dx$

4. Let  $g(x) = \int_4^{x^2} \cos(t^3) dt.$

a) Determine  $g(2).$

b) Determine  $g'(x).$

c) Given that  $g(1) \approx .1393$  and  $g(3) \approx -.0185$ , determine the approximate value of

$$\int_1^9 \cos(t^3) dt.$$

5. The graph of  $y = \sin^2 x$  on the interval  $[0, \pi/2]$  is given below.

a) True or false:

$$\int_0^{\pi/2} \sin^2 x dx = \frac{1}{3} \sin^3 x \Big|_0^{\pi/2}$$

b) True or false:

$$\int_0^{\pi/2} \sin^2 x dx = \frac{1}{3 \cos x} \sin^3 x \Big|_0^{\pi/2}$$

c) Arguing solely by areas on the graph (don't antidifferentiate!), determine the exact value for  $\int_0^{\pi/2} \sin^2 x dx$ .

6. The graph of  $y = x^2(8 - x)$  is given below. Let  $M_n$  denote the Riemann sum which approximates the area of the region bounded by the graph and the  $x$ -axis, using  $n$  equal-sized subintervals, evaluating at the **midpoints** of the subintervals. (Or in other words,  $M_n$  is the midpoint rule approximation.)

a) On the graph, sketch the rectangles corresponding to  $M_4$ .

b) Evaluate  $M_4$ .

c) Which of the following sums represents the correct expression for  $M_n$ .

I)  $\sum_{i=1}^n \frac{64i^2}{n^2} \left(8 - \frac{8i}{n}\right) \frac{8}{n}$

III)  $\sum_{i=1}^n \frac{(8i - 4)^2}{n^2} \left(8 - \frac{8i - 4}{n}\right) \frac{8}{n}$

II)  $\sum_{i=1}^n \frac{64i^2}{n^2} \left(8 - \frac{8i}{n}\right) \frac{4}{n}$

IV)  $\sum_{i=1}^n \frac{(8i - 4)^2}{n^2} \left(8 - \frac{8i - 4}{n}\right) \frac{4}{n}$

d) What is the exact value of

$$\lim_{n \rightarrow \infty} M_n?$$

