MATH 12 **Name:**_____

Calculus with Analytic Geometry II

Final

To whom it may concern: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$.

Show all work.

1. Find the derivatives $\frac{dy}{dx}$ for:

   a) $y = e^{\sin x}$

   b) $y = \int_2^{2x} \sqrt{t^3+1}dt$

   c) $y = x^{\tan^{-1} x}$

2. On the grid provided below, sketch the graph of a function with the following properties:

   a) $f(x)$ is continuous on its domain $[0, 6]$.

   b) $f(x)$ has an inverse function $g(x)$.

   c) $f(0) = 5$; $f(3) = 2$; $f(6) = 1$;

   d) $\int_0^6 f(x)dx = 13$ (at least roughly).

   e) If you had another such function without the value of $\int_0^6 f(x)dx$ specified, how large could the value of $\int_0^6 f(x)dx$ be?

3. Let R be the region bounded by graphs of $y = \ln x$, $x = e$, and $y = 2$. Express the area of R as an integral with respect to dx or dy and evaluate the area. (Hint: the dy integral is easier to evaluate, but either one is doable.)

4. Referring to the previous question, write down two different integral expressions for the volume of the solid generated by rotating R about the y-axis. DO NOT EVALUATE.

5. Write down the Trapezoid Rule and Simpson's rule approximation with $n = 6$ to the integral $\int_2^5 \frac{dx}{\ln x}$. It is not necessary to calculate the exact decimal value of the approximations.

6. Each of the following integrals is assigned a point total. Do at least 30 points worth of integrals. Do not attempt more than 50 points worth. SHOW ALL WORK. (Calculator generated approximations are worth nothing.)

a) (10) $\int_0^{\frac{\pi}{3}} \sin^5\theta\cos^4\theta d\theta$

b) (10) $\int_1^3 \frac{5x^2-7x-4}{x^2(2x+1)}dx$

c) (10) $\int \tan^4\theta\sec^4\theta d\theta$

d) (10) $\int x^2e^{-x}dx$

e) (15) $\int \frac{dx}{\sqrt{x^2+16}}$

f) (25) $\int x\cos^{-1}xdx$

g) (25) $\int_0^4 x^2\ln(x^2+9)dx$

7. Express each of the following improper integrals as a limit. Carefully evaluate the limits which exist.

a) $\int_{\frac{1}{3}}^{\frac{2}{3}} \frac{dx}{\sqrt{3x-1}}$

b) $\int_0^\infty x^2e^{-x^3}dx$

8. True or false:

$$\int \frac{e^{2x}dx}{\sqrt{1+e^x}} = \frac{2}{3}(e^x-2)\sqrt{1+e^x}+C$$

Explain your answer.

9. Evaluate $\int \cot^2 x\csc xdx$ by filling in the following details:

a) Substitute $u=\cos x$. Show that the resulting integral becomes

$$\int \frac{-u^2du}{(1-u^2)^2}.$$

b) Determine the partial fraction expansion of $\frac{-u^2}{(1-u^2)^2}$ and integrate it.