MATH 13 **Name:**_____

Calculus with Analytic Geometry III

Final **08 June 2011**

1. Let $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ and $\mathbf{w} = -2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

   Find

   a) $|\mathbf{u}|$.

   b) $\mathbf{v} \times \mathbf{w}$.

   c) The triple scalar product of $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.

   d) The angle between $\mathbf{u}$ and $\mathbf{w}$.

   e) Any vector at right angles to $\mathbf{u}$

2. Let P be the plane $2x - 5y + 4z = 20$.

   a) Find the equation for the line L perpendicular to P at the point $(1, -2, 2)$

   b) Find the equation for the plane containing L and the point $(1, 1, 1)$.

3. Find the area of the triangle with vertices $(1, 1, 1)$, $(2, 3, 4)$ and $(3, 4, -1)$.

4. Let $\mathbf{R}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} + t^2\mathbf{k}$ give the position of a particle at time t.

   a) Find the velocity vector of the particle at time t.

   b) Find the acceleration vector of the particle at time t.

   c) Find the speed of the particle at time t. Simplify your answer.

   d) Calculate the vectors $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$ for this path at time $t = \pi/4$.

   e) Calculate the curvature of the path at time $t = \pi/4$.

   f) Find the distance traveled by the particle from time $t = 0$ to time $t = 2$.

5. Four of the following equations correspond to four of the five graphs shown. One graph is extraneous. Match the four equations to the graphs, and sketch the graph of the equation whose graph is not shown.

$$z = x^2 - y^2; \quad 9x^2 + 4y^2 + z^2 = 36; \quad z = 18 - x^2 - 9y^2; \quad z^2 - \frac{x^2}{4} - y^2 = 1; \quad \frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{4} = 1$$

6. Determine whether the following series converge or diverge. For **three** of the series give a brief, but thorough justification of your answer.

$$\text{a) } \sum_{n=2}^{\infty} \frac{n+1}{n^3+1}$$

$$\text{b) } \sum_{n=2}^{\infty} \frac{1}{n \ln n}.$$

$$\text{c) } 1 + 1/2 - 1/3 + 1/4 + 1/5 - 1/6 + 1/7 + 1/8 - 1/9 + \cdots$$

$$\text{d) } 1 + 1/2^2 - 1/3^2 + 1/4^2 + 1/5^2 - 1/6^2 + 1/7^2 + 1/8^2 - 1/9^2 \cdots$$

$$\text{e) } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}}.$$

7. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x-3)^n}{\sqrt{n^2+1}}$$

8. Let $f(x)$ be an infinitely differentiable function at $x = a$

   a) Give the definition of the Taylor Series associated to $f(x)$ centered at a.

   b) Give the Taylor series for e^x centered at $x = 0$.

   c) Give the Taylor series for te^{-t^3} centered at $t = 0$.

   d) Give the Taylor series for
   $$\int_0^x te^{-t^3} dt.$$

   e) Give a series expression for the integral
   $$\int_0^1 te^{-t^3} dt.$$

   f) Use the previous series to approximate the integral correctly to 4 decimal places. You must show your work on this part to receive credit.

9. Suppose a particle travels a path in such a way that its velocity at some moment is given by $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and its acceleration is given by $3\mathbf{i} - 5\mathbf{k}$. Is the particle speeding up or slowing down at that moment? What is the rate of change of the speed at that moment?

10. Sketch two of the following graphs:

$$\frac{x^2}{9} + y^2 = 1; \quad \frac{x^2}{9} - y^2 = 1; \quad \frac{x}{9} + y^2 = 1$$