MATH 21 **Name:**____________________________________

Calculus with Analytic Geometry IV

Midterm I **1 February 2002**

1. Determine whether the series converges or diverges. If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{3^n}{e^n}.$$

$$\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)} \right).$$

2. a) State the Alternating Series test and use it to show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$$

converges.

b) Let

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} \quad \text{and} \quad S_N = \sum_{n=1}^{N} \frac{(-1)^n}{\sqrt{n^2+1}}.$$

(S_N is the Nth partial sum of S.) Write the following 6 numbers in order from smallest to largest: $S_1, S_{10}, S_{100}, S_{2001}, S_{2002}, S$. Explain your answer.

3. Determine whether the following series converge or diverge. Give a brief but thorough justification of your answer.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$. b) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt[3]{n^2+3}}{\sqrt{n^3-2}}$.

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)!}{(n!)^2}$. d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+\cos n}$.

e) Let $\{d_n\}_{n=1}^{\infty}$ be the sequence defined by the decimal expansion of $e = 2.718281\ldots = 2.d_1d_2d_3d_4d_5d_6\ldots$. (So $d_1 = 7$, $d_2 = 1$, ...)

$$\sum_{n=1}^{\infty} \frac{(-1)^{d_n}}{n\sqrt{n}}.$$

4. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^2}$$