

MATH 13
Calculus with Analytic Geometry III
Final Practice

Name: _____

1. Let $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{k}$. Find
 - a) $|\mathbf{v}|$.
 - b) The unit vector \mathbf{u} in the direction of \mathbf{v} .
 - c) $\mathbf{v} \times \mathbf{w}$
 - d) Any vector which is orthogonal to $\mathbf{v} + \mathbf{w}$.
2. Let $\mathbf{R}(t) = e^t\mathbf{i} + e^t \sin t\mathbf{j} + e^t \cos t\mathbf{k}$ give the position of a particle at time t .
 - a) Find the velocity vector of the particle at time t .
 - b) Find the speed of the particle at time t .
 - c) Find the vectors \mathbf{T} , \mathbf{N} , \mathbf{B} at $t = 0$.
 - d) Find the distance traveled by the particle from time $t = 0$ to time $t = 2$.
3. Find the equation for the plane through the line $x = t + 2$, $y = 2t - 3$, $z = 7t + 5$ and the point $(-1, 4, -3)$
4. Find the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 2, 3)$.
5. Let $f(x)$ be an infinitely differentiable function at $x = c$
 - a) Give the definition of the Taylor Series associated to $f(x)$ centered at c .
 - b) Give the Taylor series for e^x centered at $x = 0$.
 - c) Give the Taylor series for te^{-t^3} centered at $t = 0$.
 - d) Give the Taylor series for
$$\int_0^x te^{-t^3} dt.$$
 - e) Give a series expression for the integral
$$\int_0^1 te^{-t^3} dt.$$
 - f) How many terms (starting from the first term) in the series in part d) should be summed to obtain an approximation correct to 3 decimal places.
6. Give an equation for the circle in the xy -plane containing $(2, 4)$ and $(4, 6)$ as diametrically opposite points.
7. Sketch the following graphs in the xy -plane:

$$x = 4 - y^2; \quad \frac{x^2}{9} + y^2 = 1; \quad ; \quad \frac{x^2}{9} - y^2 = 1$$

8. Sketch the following graphs in xyz -space:

$$x = 4 - y^2; \quad z = y^2 - x^2$$