

Problem Solving Fall 2007

Problem Set #2

1. For each non-empty subset of $\{1, 2, 3, 4, 5, 6, 7\}$ arrange the members in decreasing order with alternating signs and take the sum. For example, for the subset $\{5\}$ we get 5. For $\{6, 3, 1\}$ we get $6 - 3 + 1 = 4$. Find the sum of all the resulting numbers.

Can you generalize your answer?

2. Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x) + \ln(x+3)}} dx$$

(From the 1987 Putnam)

3. Find all numbers between 60 and 70 which are exact divisors of $2^{48} - 1$.
4. A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form $(a\sqrt{b} + c)/d$, where a, b, c and d are integers. (From the 1989 Putnam.)