

Problem Solving Fall 2007

Problem Set #3

1. Place a knight on each square of a 7x7 chessboard. Is it possible for each knight to simultaneously make a legal move so that each knight ends up in its own square?
2. Remove two opposite corners on an 8x8 chessboard. Can the remaining board be covered with 31 1x2 dominoes?.
3. Remove one corner of a 8x8 chessboard. Can the remaining board be covered with 21 1x3 dominoes?
4. If 21 1x3 dominoes are used to tile an 8x8 chessboard, which squares can be left uncovered?
5. (Putnam 1997– A5 !) Let N_n denote the number of ordered n -tuples of positive integers (a_1, a_2, \dots, a_n) such that

$$1/a_1 + 1/a_2 + \dots + 1/a_n = 1.$$

Determine whether N_{10} is even or odd.

6. (Putnam 1991) A 2x3 rectangle has vertices at $(0, 0)$, $(2, 0)$, $(0, 3)$ and $(2, 3)$. It then rotates 90° clockwise about the point $(5, 0)$, then 90° clockwise about the point $(7, 0)$, and finally, 90° clockwise about the point $(10, 0)$. (The side originally on the x -axis is now back on the x -axis.) Find the area of the region above the x -axis and below the curve traced out by the point whose initial position is $(1, 1)$.