

# Problem Solving Fall 2007

## Problem Set #6

1. Show that out of any six people, you can find three people who are all mutual acquaintances or are all mutual strangers. Is the same conclusion true for any group of 5 people?
2. Each segment joining two vertices of a regular hexagon is colored either blue or red. Is there necessarily a monochromatic triangle? Two monochromatic triangles? Three monochromatic triangles?
3. Let  $n$  be a positive integer, and define

$$f(n) = 1! + 2! + 3! + \cdots n!.$$

Find polynomials  $P(x)$  and  $Q(x)$  such that

$$f(n+2) = P(n)f(n+1) + Q(n)f(n)$$

for all  $n \geq 1$ .

4. A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?