

# Problem Solving Fall 2007

## Problem Set #7

1. Can the numbers of dots on a pair of fair dice be changed so that the chances of rolling 2, 3,  $\dots$ , 12 are the same as with regular dice?
2. A knight on a 5x5 chessboard moves randomly, choosing any legal square with uniform probability. In the long run, what fraction of its time is spent in the central square. (A knight is allowed to move two squares up/down or left/right and one square in the perpendicular direction.)
3. Two unequal numbers are chosen uniformly from all integers between 1 and  $N$ . What is the expected value of their product.
4. Ten percent of the surface of a sphere is colored white, the rest black. Show that there is a cube inscribed in the sphere with all black vertices.