Math 12 Practice Problems

These problems were compiled by Ed Schaefer. They should not be interpreted to indicate what will be on my tests in any way, but they can give you some practice working on new problems. Unlike an exam, I have ordered the problems so that, to a large degree, they follow the order of the class. This allows you to stop when you encounter material we haven't covered yet. Some problems have hints. If you need the hints, they are between the questions and the answers.

Questions:

1. Find the area above the x-axis, to the left of $x = 8$, to the right of $x = 5$ and below $y = 2x+4$.

2. Let $A(x)$ give the area under $y = \sin(t)$ from $t = 0$ to $t = x$. Find $A'(\pi/2)$.

3. Compute the following:

$$a)\ \int_2^3 \frac{1}{(t-1)^3}\,dt \qquad b)\ \int_1^2 \sin(\pi x + 1)\,dx \qquad c)\ \int_0^{\pi/4} \cos(5x)\sin(5x)\,dx$$

$$d)\ \int_0^{\pi/16} 7\sec(4x)\tan(4x)\,dx \qquad e)\ \int_1^2 \frac{1}{x^2\sqrt{1+\frac{1}{x}}}\,dx \qquad f)\ \int_0^1 \frac{x}{(3+2x^2)^4}\,dx$$

$$g)\ \int_1^7 \sqrt{29-4x}\,dx \qquad h)\ \frac{d}{dy}\int_0^y \cos(m^2+3)\,dm \qquad i)\ \frac{d}{dx}\int_1^{x^2} \frac{1}{t^4+3}\,dt$$

4. Approximate the area under $y = 10 - x^2$ on the interval $[1,3]$ using 4 subintervals and midpoints.

5. Find the area bounded by the curves $y = x^4 - 4x^2$ and $y = 5x^2$.

6. Find the number c such that the line $x = c$ divides the region bounded by $y = \sqrt{x}$, $x = 16$ and the x-axis into two regions of equal area.

7. Evaluate $\int_{-3}^{3} 10 - \sqrt{9-x^2}\,dx$ by interpreting it in terms of area.

8. Use calculus to find the volume of the cone of height 1 whose base has radius 1.

9. Find the volume gotten by spinning region bounded by $y = x^2 - x$ and the x-axis about a) the x-axis, b) $y = 1$, c) the y-axis, d) $x = 2$.

10. Find the average value of $\sin^2(x)$ on $0 \le x \le \pi$.

11. A 3 centimeter piece of wire has density $\delta(x) = 1 + x^2$ grams per centimeter at the point x centimeters from the left end. Where is the center of mass?

12. If $6\ln(3+x) = 7$ then find x.

13. Find the area enclosed by $y = e^{-x}$, $x = 0$, $y = 0$ and $x = -3$.

14. $\frac{d}{dx}\frac{e^{3x}\sqrt{2-x}}{x^2}$

15. $\frac{d}{dx}(\sqrt{x})^x$

16. $\frac{d}{dx}\ln(\ln(x))$

17. Compute $\frac{d}{dx}3\sinh(3/x)$

18. Simplify $\cosh(\ln(2))$.

19. Calculate the following definite integrals and limits (Hint set 1)

$$a.\ \int_0^1 xe^{x^2}\,dx \quad b.\ \int_0^{\pi/4}\frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx \quad c.\ \int_1^e \frac{1}{x}\cos(\ln(x))\,dx \quad d.\ \int_0^{\pi/9}\tan(3x)\,dx$$

$$e.\ \int_0^7 10e^{-3t}\,dt \quad f.\ \int_1^2 \frac{1+y+y^2}{y}\,dy \quad g.\ \int_0^1 \frac{2}{4x+3}\,dx \quad h.\ \int_0^1 \frac{2}{(4x+3)^2}\,dx$$

$$i.\ \int_2^3 \frac{xdx}{\sqrt{x^2-1}} \quad j.\ \int_2^3 \frac{1}{\sqrt{x^2-1}}\,dx \quad k.\ \int_5^{10}\frac{dx}{\sqrt{x-1}} \quad l.\ \int_0^{3/5}\frac{xdx}{\sqrt{1-x^2}} \quad m.\ \int_{-1/2}^{1/2}\frac{dx}{\sqrt{1-x^2}}$$

$$n.\ \int_0^1 \frac{xdx}{1+x^2} \quad o.\ \int_{-1/3}^{1/3}\frac{dx}{1+9x^2} \quad p.\ \int_0^1 \frac{xdx}{1+x^4} \quad q.\ \int_0^1 \frac{x^3dx}{1+x^4} \quad r.\ \int_0^{\sqrt{3}}\frac{dx}{9+x^2}$$

$$s.\ \int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx \quad t.\ \int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx \quad u.\ \int_0^1 \frac{x+1}{x^2+1}\,dx \quad v.\ \int_0^4 \sqrt{16-x^2}\,dx$$

$$w.\ \lim_{x\to\frac{\pi}{2}^-}\sec(x)-\tan(x) \quad x.\ \lim_{x\to\infty}\frac{\ln(x)}{x} \quad y.\ \lim_{x\to 0}\frac{x\sin(\text{x})-x^2}{2\cos(x)-2+x^2}$$

$$z.\ \int_0^{\pi/2}\sin^4(x)\cos^3(x)\,dx$$

20. Determine the following indefinite and definite integrals (Hint set 2)

$$a.\ \int\frac{dx}{\sqrt{4-x^2}} \quad b.\ \int\frac{xdx}{\sqrt{4-x^2}} \quad c.\ \int\frac{x^2dx}{\sqrt{4-x^2}} \quad d.\ \int\frac{xdx}{\sqrt{4-x}} \quad e.\ \int\frac{dx}{\sqrt{x^2-4}} \quad f.\ \int\frac{dx}{x\sqrt{x^2-4}}$$

$$g.\ \int\frac{dx}{\sqrt{4x^2+1}} \quad h.\ \int\frac{xdx}{\sqrt{4x^2+1}} \quad i.\ \int x^3\sqrt{x^2+4}dx \quad j.\ \int\frac{dx}{x^2+4} \quad k.\ \int\frac{xdx}{x^2+4}$$

$$l.\ \int\frac{x^2dx}{x^2+4} \quad m.\ \int\frac{x^4dx}{x^2-4} \quad n.\ \int\frac{dx}{4x^2-1} \quad o.\ \int\frac{x^2dx}{(x+4)^3} \quad p.\ \int\frac{dx}{x^3+4x}$$

$$q.\ \int\sin(3\sqrt{x})dx \quad r.\ \int\frac{\ln(x)dx}{\sqrt{x}} \quad s.\ \int(x+1)\cos(3x)dx \quad t.\ \int x^2e^{-x}dx$$

$$u.\ \int\tan^5(x)\sec^5(x)dx \quad v.\ \int\sin^5(x)\cos^5(x)dx \quad w.\ \int\tan^4(x)\sec^4(x)dx \quad x.\ \int\cos^2(x)dx$$

$$y.\ \int_0^1\ln(x)dx \quad z.\ \int_0^2\frac{dx}{x^2-1} \quad aa.\ \int_2^4\frac{dx}{x^2-1} \quad ab.\ \int_4^\infty\frac{dx}{x^2-1} \quad ac.\ \int_0^\infty e^{-x/2}dx$$

21. Solve $\frac{dy}{dx} = xy^2$.

22. Find the function $f(x)$ with the property that its derivative is x times itself and $f(0) = 2$.

23. Write down Simpson's rule and Trapezoid rule approximations to $\int_0^1 \frac{\sin(x)}{x} dx$ with $n = 6$.

24. On 9/1/07, there were 50000 stations playing Westlife. On 11/1/07 there were 30000. The rate of decay is proportional to the number of stations playing them. When will there be 5000 stations left (and it will be safe to turn the radio on again)?

Hint sets:

1. $a.u = x^2$, $b.u = \cos(x) + \sin(x)$, $c.u = \ln(x)$, $d.u = \cos(3x)$, $g, h.u = 4x + 3$, $i.u = x^2 - 1$, $j.\cosh^{-1}$, $k.u = x - 1$, $l.u = 1 - x^2$, $m.\sin^{-1}$, $n.u = 1 + x^2$, $o.u = 3x$, $p.u = x^2$, $q.u = 1 + x^4$, $s.\sinh^{-1}$, $t.u = 1 + x^2$, u. break into 2 frac'ns, v. circle.

2. a. $\sin^{-1}(u)$, b. $u = 4 - x^2$, c. $x = 2\sin(\theta)$, d. parts, e. $\cosh^{-1}(u)$, f. $x = 2\sec(\theta)$, g. $\sinh^{-1}(u)$, h. $u = 4x^2 + 1$, i. $x = 2\tan(\theta)$ or $w = x^2$ then parts, j. $\tan^{-1}(u)$, k. $u = x^2 + 4$, l,m. long div'n, m-p. part'l frac'ns, q. $w = 3\sqrt{x}$ then parts, r-t. parts, z. $\int_0^1 + \int_1^2$, ab. write antid as single log.

Answers:

1. 51

2. 1

3. $a)$ 3/8 $b)$ $-2\cos(1)/\pi$ $c)$ 1/20 $d)$ $\frac{7\sqrt{2}}{4} - \frac{7}{4}$ $e)$ $2\sqrt{2} - \sqrt{6}$
   $f)$ 49/20250 $g)$ 62/3 $h)$ $\cos(y^2 + 3)$ $i)$ $2x/(x^8 + 3)$

4. 91/8

5. 324/5

6. $32^{2/3}$

7. $60 - (9\pi/2)$

8. $\pi/3$

9. $a)$ $\pi/30$, $b)$ $11\pi/30$ $c)$ $\pi/6$ $d)$ $\pi/2$

10. $1/2$

11. $33/16$ cm from left

12. $-3+e^{7/6}$

13. e^3-1

14. $\frac{e^{3x}\sqrt{2-x}}{x^2}(3-\frac{1}{4-2x}-\frac{2}{x})$

15. $\frac{1}{2}(\ln(x)+1)(\sqrt{x})^x$

16. $\frac{1}{x\ln(x)}$

17. $-\frac{9}{x^2}\cosh(\frac{3}{x})$

18. $5/4$

19. $a.$ $\frac{1}{2}(e-1)$
$b.$ $\ln(2)/2$ $c.$ $\sin(1)$ $d.$ $\ln(2)/3$ $e.$ $\frac{10}{3}(1-e^{-21})$ $f.$ $5/2+ln(2)$
g $1/2ln(7/3)$ $h.$ $2/21$ $i.$ $\sqrt{8}-\sqrt{3}$ $j.$ $\cosh^{-1}(3)-\cosh^{-1}(2)$ $k.$ 2 $l.$ $1/5$
$m.$ $\pi/3$ $n.$ $\frac{1}{2}\ln(2)$ $o.$ $\pi/6$ $p.$ $\pi/8$ $q.$ $\frac{1}{4}ln(2)$ $r.$ $\pi/18$ $s.$ $\sinh^{-1}(1)$
$t.$ $\sqrt{2}-1$ $u.$ $\ln(\sqrt{2})+\pi/4$ $v.$ 4π $w.$ 0 $x.$ 0 $y.$ -2 $z.$ $2/35$

20. $a.$ $\sin^{-1}(x/2)+C$ $b.$ $-\sqrt{4-x^2}+C$ $c.$ $2\sin^{-1}(x/2)-\frac{x}{2}\sqrt{4-x^2}+C$
$d.$ $-2x\sqrt{4-x}-\frac{4}{3}(4-x)^{3/2}+C$ $e.$ $\cosh^{-1}(x/2)+C$ $f.$ $\frac{1}{2}\sec^{-1}(x/2)+C$
$g.$ $\frac{1}{2}\sinh^{-1}(2x)+C$ $h.$ $\frac{1}{4}\sqrt{4x^2+1}+C$ $i.$ $\frac{1}{5}(x^2+4)^{5/2}-\frac{4}{3}(x^2+4)^{3/2}+C$ $j.$ $\frac{1}{2}\tan^{-1}(x/2)+C$
$k.$ $\frac{1}{2}\ln(x^2+4)+C$ $l.$ $x-2\tan^{-1}(x/2)+C$ $m.$ $4\ln|x-2|-4\ln|x+2|+x^3/3+4x+C$
$n.$ $\frac{1}{4}\ln|2x-1|-\frac{1}{4}\ln|2x+1|+C$ $o.$ $\ln|x+4|+8/(x+4)-8/(x+4)^2+C$
$p.$ $\frac{1}{4}\ln|\text{x}|-\frac{1}{8}\ln|x^2+4|+C$ $q.$ $-\frac{2}{3}\sqrt{x}\cos(3\sqrt{x})+\frac{2}{9}\sin(3\sqrt{x})+C$ $r.$ $2\sqrt{x}\ln(x)-4\sqrt{x}+C$
$s.$ $\frac{1}{3}(x+1)\sin(3x)+\frac{1}{9}\cos(3x)+C$ $t.$ $-e^{-x}(x^2+2x+2)+C$ $u.$ $\frac{1}{9}\sec^9(x)-\frac{2}{7}\sec^7(x)+\frac{1}{5}\sec^5(x)+C$ $v.$ $\frac{1}{6}\sin^6(x)-\frac{1}{4}\sin^8(x)+\frac{1}{10}\sin^{10}(x)+C$ $w.$ $\frac{1}{5}\tan^5(x)+\frac{1}{7}\tan^7(x)+C$
$x.$ $\frac{x}{2}+\frac{1}{4}\sin(2x)+C$ $y.$ -1 $z.$ div $aa.$ $\frac{1}{2}\ln(9/5)$ $ab.$ $\frac{1}{2}\ln(5/3)$ $ac.$ 2

21. $y=2/(C-x^2)$

22. $y=2e^{x^2/2}$

23. $T_6=1/12(1+2\sin(1/6)/(1/6)+2\sin(1/3)/(1/3)+2\sin(1/2)/(1/2)+2\sin(2/3)/(2/3)+2\sin(5/6)/(5/6)+\sin(1))$, $S_6=1/18(1+4\sin(1/6)/(1/6)+2\sin(1/3)/(1/3)+4\sin(1/2)/(1/2)+2\sin(2/3)/(2/3)+4\sin(5/6)/(5/6)+\sin(1))$

24. $2\ln(.1)/\ln(3/5)$ (≈ 9) months from 9/1/07