

PERFECT
SCORES

- 3/16 1. (15) Find $\frac{dy}{dx}$ given $y = \cos^3(5 \cos 4x^2)$.
- 5/16 2. (15) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given $y = 4x^3 \cos 2x$.
- 7/16 3. (15) What is (are) the asymptote(s) of $y = \frac{x^2 + 1}{x + 1}$? For each asymptote, give its type (i.e., *vertical*, *horizontal*, or *oblique*).
- 9/16 4. (15) (a) Use Calculus methods to find the critical values of x for the function $f(x) = 2x^3 - 9x^2 + 12x + 6$.
EASIEST (b) Determine which of these x -values corresponds a local *maximum* and indicate the corresponding y -value.
- 5/16 5. (20) An apple grower has determined that if 22 apple trees are planted per acre, the yield is 744 apples per tree. In addition, after years of experimentation, he has found that for *each* additional tree per acre, he gets 12 fewer apples per tree (e.g., 3 *more* trees per acre give 36 *fewer* apples per tree)! Using Calculus methods, determine how many trees should be planted per acre to obtain the maximum crop of apples.
- 1/16 6. (20) A metal can is to be made to hold 1 liter of oil. When making the cylindrical side of the can, there is no wasted metal, but the top and bottom circular pieces must be cut from squares of material and the excess is wasted. Thus, the cost of material must take into account that the end pieces of the can are made from squares, rather than from circles. On the other hand, the volume is computed assuming the top and bottom are circles. Find the dimensions (i.e., of the *radius* and the *height*) of the can that will minimize the cost of the metal to manufacture it.
HARDEST

REMEMBER!

- ⇒ Please start EACH problem on a NEW PAGE SIDE!
- ⇒ PLEASE *label* (or underline or box in) all ANSWERS clearly.
- ⇒ Remember to show all work for full credit.
- ⇒ ALSO remember, NO CALCULATORS!

Thanks.

STATS

H1 - 94	MEDIAN - (73.5)
LO - 26	MEAN - 65.25
# TESTS - 16	σ - 19.75

1. $y = \cos^3(5 \cos 4x^2) \Rightarrow \frac{dy}{dx} = \frac{3 \cos^2(5 \cos 4x^2) (-\sin(5 \cos 4x^2)) (-5 \sin 4x^2) (8x)}{1}$


2. $y = 4x^3 \cos 2x \Rightarrow \frac{dy}{dx} = -4x^3(\sin 2x)2 + (\cos 2x)12x^2$
 $= -8x^3 \sin 2x + 12x^2 \cos 2x$

$\Rightarrow \frac{d^2y}{dx^2} = -8x^3(\cos 2x)2 + (\sin 2x)(-24x^2) + 12x^2(-\sin 2x)2 + (\cos 2x)24x$
 $= -16x^3(\cos 2x) - 48x^2 \sin 2x + 24x \cos 2x$

3. $y = \frac{x^2+1}{x+1} \Rightarrow x+1 \overline{) \begin{array}{r} x^2+0x+1 \\ x^2+x \\ \hline -x+1 \\ -x-1 \\ \hline 2 \end{array}}$
 \Downarrow
 $y = x-1 + \frac{2}{x+1}$
 vert asympt at $x=-1$
 oblique asympt at $y=x-1$

4. (a) $y = 2x^3 - 9x^2 + 12x + 6$
 $\frac{dy}{dx} = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-2)(x-1)$
 setting $\frac{dy}{dx} = 0 = 6(x-2)(x-1)$
 $\Rightarrow x=2$ or 1 are critical values
 (b) $y|_{x=1} = 2 - 9 + 12 + 6 = 11$
 $y|_{x=2} = 2 \cdot 8 - 9 \cdot 4 + 24 + 6 = 10$
 \Rightarrow max is at $x=1$ when $y=11$

5. 22 trees \leftrightarrow 744 apples/tree
 $22+1x \leftrightarrow 744-12x$
 crop = (trees)(apples/tree)
 $= (22+x)(744-12x)$
 $= 22 \cdot 744 + 480x - 12x^2$
 $\frac{dC}{dx} = 480 - 24x$
 setting equal to zero
 $\Rightarrow 24x = 480$
 $x = \frac{480}{24} = 20$
 \therefore # of trees = $22+20 = 42$

6.  $V = 1$ liter
 $A_{\text{Tot mat}} = 2\pi r h + 2(\pi r^2)$
 $= 2\pi r \left(\frac{1}{\pi r^2} \right) + 2\pi r^2$
 $= 2r^{-1} + 2\pi r^2$
 $\frac{dA}{dr} = 2(-1)r^{-2} + 4\pi r$
 $= -\frac{2}{r^2} + 4\pi r$
 $= \frac{-2 + 4\pi r^3}{r^2} = 0$
 $4\pi r^3 = 2$
 $r^3 = \frac{1}{2\pi}$
 $r = \sqrt[3]{\frac{1}{2\pi}}$
 $V = \pi r^2 h$
 $h = \frac{V}{\pi r^2} \Big|_{V=1} = \frac{1}{\pi r^2}$
 $h = \frac{1}{\pi \left(\frac{1}{2\pi} \right)^2} = \frac{4}{\pi}$