Math 12 (sect. 44953) (9:15 AM)—Winter, 2009 Midterm III (45 minutes)
D.C. Smolarski, S.J. Monday, February 23, 2009

*NOTES:*

1. Problems can be written in the blue book IN ANY ORDER, but please START each problem on a NEW PAGE (EITHER side) and label it properly.
2. PLEASE *label* (or underline or box in) all ANSWERS clearly.
3. **NO CALCULATORS!**

#perfect SCORES

4/31 "HARDEST"

1. (16) The area between the lines $y = e^{2x}$, $x = 1$, and $x = 2$ is rotated around the $x$-axis. Find the volume of the resulting solid.

18/31

2. (13) Find $\frac{dy}{dx}$ if $y = \cos\left(e^{\sin^2 3x^2}\right)$.

7/31

3. (16) Evaluate $\int \frac{1-x}{\sqrt{1-x^2}}dx$.

19/31

4. (16) Evaluate $\int \frac{\sin x \cos x}{\cos^2 x + 1}dx$.

20/31 EASIEST

5. (13) Evaluate $\int \frac{\sin x}{\cos^2 x + 1}dx$.

17/31

6. (13) Evaluate $\int x^2 \cos x dx$.

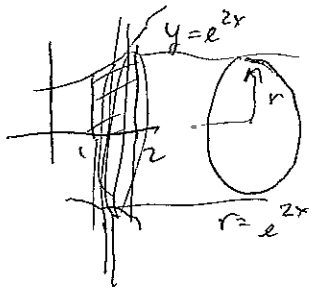
14/31

7. (13) Evaluate $\int x \tan^2 3x^2 dx$.

STATS

| | | |
|---|---|---|
| HIGH SCORE | 100 | # EXAMS 31 |
| LOW SCORE | 44 | |
| MEDIAN | 72 | |
| MEAN | 73.516 | |

MT 12 - MID III - W09 - 915a

1. 


$V = \pi\int_1^2 (e^{2x})^2 dx = \pi\int_1^2 e^{4x}dx$ $\left[\begin{array}{l} u=4x \\ \frac{du}{dx}=4 \\ \frac{du}{4}=dx \end{array}\right.$

$= \frac{\pi}{4}\int_{x=1}^{2} e^u du = \frac{\pi}{4}e^u\Big]_{x=1}^{2} = \frac{\pi}{4}e^{4x}\Big]_1^2 = \underline{\frac{\pi}{4}(e^8 - e^4)}$

2. $y = \cos(e^{\sin^2 3x^2}) \Rightarrow \frac{dy}{dx} = -\sin(e^{\sin^2 3x^2})\cdot e^{\sin^2 3x^2}\cdot 2\sin 3x^2\cdot\cos 3x^2\cdot 6x$

3. $\int\frac{1-x}{\sqrt{1-x^2}}dx = \int\frac{dx}{\sqrt{1-x^2}} - \int\frac{x\,dx}{\sqrt{1-x^2}} = \arcsin x - \int\frac{x\,dx}{\sqrt{1-x^2}}$ $\left[\begin{array}{ll} u=1-x^2 & \frac{du}{-2}=x\,dx \\ \frac{du}{dx}=-2x & \end{array}\right.$

$= \arcsin x - \left(\frac{-1}{2}\right)\int\frac{du}{\sqrt{u}} = \arcsin x + \frac{1}{2}\int u^{-1/2}du = \arcsin x + \frac{1}{2}\frac{u^{1/2}}{1/2} + C$

$= \arcsin x + \sqrt{1-x^2} + C$

4. $\int\frac{\sin x\cos x}{\cos^2 x+1}dx$ $\left[\begin{array}{l} u=\cos^2 x+1 \\ \frac{du}{dx}=-2\cos x\sin x \\ -\frac{du}{2}=\cos x\sin x\,dx \end{array}\right]$ $= -\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln|u|+C$

$= -\frac{1}{2}\ln(\cos^2 x+1)+C$

5. $\int\frac{\sin x\,dx}{\cos^2 x+1}$ $\left[\begin{array}{l} u=\cos x \\ \frac{du}{dx}=-\sin x \\ -du=\sin x\,dx \end{array}\right]$ $= -\int\frac{du}{u^2+1} = -\arctan u + C$

$= -\arctan(\cos x)+C$

6. $\int x^2\cos x\,dx = x^2\sin x - 2\int x\sin x\,dx = x^2\sin x - 2\left[-x\cos x + \int\cos x\,dx\right]$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

| | | | |
|---|---|---|---|
| $u=x^2$ | $dv=\cos x\,dx$ | $u=x$ | $dv=\sin x\,dx$ |
| $du=2x\,dx$ | $v=\sin x$ | $du=dx$ | $v=-\cos x$ |

7. $\int x\tan^2 3x^2\,dx$ $\left[\begin{array}{l} u=3x^2 \\ \frac{du}{dx}=6x \\ \frac{du}{6}=x\,dx \end{array}\right]$ $= \frac{1}{6}\int\tan^2 u\,du = \frac{1}{6}\int\sec^2 u - 1\,du = \frac{1}{6}(\tan u - u)+C$

$= \frac{1}{6}\tan 3x^2 - \frac{1}{6}3x^2 + C$

$= \frac{\tan 3x^2}{6} - \frac{x^2}{2} + C$