Solution to Quiz 4. Find the volume bounded by the surfaces \( y = x^2, \)
\( z = 0 \) and \( y + z = 1. \)

The planes \( z = 0 \) and \( y + z = 1 \) meet where \( y = 1 \) in the plane \( z = 0. \) So that’s the line \( y = 1 \) in the \( xy \)-plane. This gives us our shadow in the \( xy \)-plane: the region between \( y = x^2 \) and \( y = 1. \) We note that over this region, \( y + z = 1 \) or \( z = 1 - y \) is higher than \( z = 0. \) So \( z \) goes from 0 to \( 1 - y. \) For the shadow, positive \( y \) arrows enter on \( y = x^2 \) and exit on \( y = 1. \) The extreme \( x \)-values are where \( y = x^2 \) meets \( y = 1, \) i.e. at \( x = \pm 1. \) So we have volume =

\[
\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz\,dy\,dx = \int_{-1}^{1} \int_{x^2}^{1} 1 - y \, dy\,dx = \int_{-1}^{1} y - \frac{y^2}{2} \bigg|_{x^2}^{1} \, dx
\]

\[
\int_{-1}^{1} \left[ 1 - \frac{1}{2} \right] - \left[ x^2 - \frac{x^4}{2} \right] \, dx = \frac{1}{2} \int_{-1}^{1} x - \frac{x^3}{3} + \frac{x^5}{10} \, dx
\]

\[
= \left[ \frac{1}{2} - \frac{1}{3} + \frac{1}{10} \right] - \left[ \frac{1}{2} + \frac{1}{3} - \frac{1}{10} \right] = \frac{8}{15}.
\]