Quiz 2. For \( f(x, y) = \frac{1}{2x+y+1} \) you leave \((0, 1)\) in the direction \(3i + 4j\). About how far would you have to go for \( f \) to decrease by 1?

Solution. Let’s find the directional derivative of \( f \) at \((0, 1)\) in the direction of \(3i + 4j\). That’s denoted \( D_u f(0, 1) \) where \( u \) is the unit vector pointing in the same direction as \(3i + 4j\). That can be computed by \( \vec{\nabla} f(0, 1) \cdot \vec{u} \).

Let’s first find \( \vec{\nabla} f = f_x i + f_y j \). Now \( f = (2x + y + 1)^{-1} \) so \( f_x = -1(2x + y + 1)^{-2}(2) = \frac{-2}{(2x+y+1)^2} \). Also \( f_y = -1(2x + y + 1)^{-2}(1) = \frac{-1}{(2x+y+1)^2} \). So \( \vec{\nabla} f = \frac{-2}{(2x+y+1)^2} i + \frac{-1}{(2x+y+1)^2} j \) and \( \vec{\nabla} f(0, 1) = -\frac{1}{2} i - \frac{1}{4} j \).

Now \( u \) is the unit vector pointing in the same direction as \(3i + 4j\). Since \(|3i + 4j| = \sqrt{3^2 + 4^2} = 5\) then \( u = \frac{3}{5} i + \frac{4}{5} j \). Now \( D_u f(0, 1) = \vec{\nabla} f(0, 1) \cdot u = (-\frac{1}{2} i - \frac{1}{4} j) \cdot (\frac{3}{5} i + \frac{4}{5} j) = (-\frac{1}{2})(\frac{3}{5}) + (-\frac{1}{4})(\frac{4}{5}) = -\frac{1}{2} \).

Now how far do we have to go in order for the function to decrease by about 1? Answer 1: The directional derivative is \(-\frac{1}{2}\). So that means for every 1 you move, the function decreases about \(\frac{1}{2}\), so move 2. Answer 2: We let \( s \) measure distance moved from \((0, 1)\) in the direction \(u\). Then \(-\frac{1}{2} = \frac{df}{ds} \approx \frac{\Delta f}{\Delta s} = \frac{-1}{\Delta s} \). So \(-\frac{1}{2} \approx \frac{-1}{\Delta s} \). Thus \(\Delta s \approx 2\).