Review of parametrizations

A curve is intrinsically a 1-dimensional object, so there should be a way to describe a curve using one variable. Such a description is called a parametrization. We usually use the variable t to describe a curve.

There are three parametrizations I expect you to memorize.

1. The line segment from (a, b, c) to (d, e, f) is parametrized by
$x = a + (d - a)t$
$y = b + (e - b)t$
$z = c + (f - c)t$, for $0 \leq t \leq 1$. Since each of the three functions of t is linear, we expect this to give us a line. Plug in $t = 0$ and get $(x, y, z) = (a, b, c)$. Plug in $t = 1$ and get $(x, y, z) = (d, e, f)$.

2. Ellipses (and circles) in the xy-plane with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are parametrized by $x = a\cos(t)$, $y = b\sin(t)$ for $0 \leq t \leq 2\pi$. Notice that you can check this works by plugging in $a\cos(t)$ for x and $b\sin(t)$ for y in $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ and confirming it gives you 1.

3. The graph of $y = f(x)$ in the xy-plane can be parametrized by letting $x = t$ and then following your nose. So if $x = t$ then $y = f(t)$. Note that letting one variable be t and following your nose can work for more general examples. For example, parametrize the curve which is the intersection of the surfaces $x + y^2 + 2z = 7$ and $y = z$ between the points $(4, 1, 1)$ and $(-1, 2, 2)$. We can experiment with letting $x = t$, but that ends up messy. So let's try letting $y = t$. Then clearly $z = t$. Plugging those into $x + y^2 + 2z = 7$ we get $x + t^2 + 2t = 7$ or $x = 7 - t^2 - 2t$. Note that since $t = y$ we have $t = 1$ at the first point and $t = 2$ at the second point. So our curve is parametrized by $x = 7 - t^2 - 2t$, $y = t$, $z = t$ for $1 \leq t \leq 2$.