Solution to quiz 1. Use polar coordinates to describe the region: $x^2 + y^2 \leq 4$, $y \geq 1$, $x \geq 0$. The solution should be of the form $f(\theta) \leq r \leq \#$ and $\# \leq \theta \leq \#$.

Solution. Think of arrows emanating from the origin. The distance each has travelled from the origin is described by r. These arrows would enter the region hitting $y = 1$ and exit the region hitting $x^2 + y^2 = 4$.

The inequality $x^2 + y^2 \leq 4$ is $r^2 \leq 4$ so $r \leq 2$. Now $y \geq 1$ becomes $r\sin(\theta) \geq 1$ or $r \geq \csc(\theta)$. So we have $\csc(\theta) \leq r \leq 2$.

What is the span of θ's over this region? The smallest θ occurs where $x^2 + y^2 = 4$ meets $y = 1$. Let's call their intersection point P. We want to find θ for P

Method 1: I drew a triangle with vertices: the origin, the point $(0, 1)$ (where $y = 1$ meets $x = 0$) and P. The vertical edge has length 1 and the hypotenuse is a radius of the circle and so has length 2. Thus the horizontal edge has length $\sqrt{3}$ and this is a famous triangle. The interior angle of the triangle at the origin is $60°$ or, in radians, $\frac{\pi}{3}$. So the θ for P (the angle between the positive x-axis and the hypotenuse) is $90° - 60° = 30°$ or in radians $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$.

Method 2. The point P, where $y = 1$ meets $x^2 + y^2 = 4$, has y-coordinate 1. We can get its x-coordinate by substituting $x^2 + 1^2 = 4$, $x = \sqrt{3}$. So $P = (\sqrt{3}, 1)$. Recall $\tan(\theta) = \frac{y}{x} = \frac{1}{\sqrt{3}}$. To figure out θ many people would draw a triangle with opposite $= 1$, adjacent $= \sqrt{3}$ and so hypotenuse $= 2$. So $\theta (= 30°) = \frac{\pi}{6}$.

The biggest θ's in the region occur on the part of $x = 0$ above the origin, namely the positive y-axis, where $\theta = \frac{\pi}{2}$. So $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$.