Solution to quiz 2. The derivative of $f(x,y)$ at $(3,4)$ in the direction toward $(4,4)$ is 2. The derivative of $f(x,y)$ at $(3,4)$ in the direction toward $(3,5)$ is -1. Find the derivative of $f(x,y)$ at $(3,4)$ in the direction toward $(2,5)$.

Solution. Let u be the unit vector pointing in the direction from $(3,4)$ toward $(2,5)$. The derivative we are looking for is denoted $D_u f(3,4)$. Now $D_u f(3,4) = \nabla f(3,4) \cdot u$. We have $\nabla f(3,4) = \frac{\partial f}{\partial x}(3,4)\mathbf{i} + \frac{\partial f}{\partial y}(3,4)\mathbf{j}$. Note $\frac{\partial f}{\partial x}(3,4)$ is the derivative of f at $(3,4)$ in the direction $\mathbf{i}$ and the vector from $(3,4)$ to $(4,4)$ is $\mathbf{i}$. So $\frac{\partial f}{\partial x}(3,4) = 2$. Similarly note $\frac{\partial f}{\partial y}(3,4)$ is the derivative of f at $(3,4)$ in the direction $\mathbf{j}$ and the vector from $(3,4)$ to $(3,5)$ is $\mathbf{j}$. So $\frac{\partial f}{\partial y}(3,4) = -1$. Thus $\nabla f(3,4) = 2\mathbf{i} - \mathbf{j}$. Now let's find u. That's the unit vector pointing in the direction from $(3,4)$ to $(2,5)$. The vector from $(3,4)$ to $(2,5)$ is $(2-3)\mathbf{i} + (5-4)\mathbf{j} = -\mathbf{i} + \mathbf{j}$, which has length $\sqrt{(-1)^2 + 1^2} = \sqrt{2}$. So $u = \frac{-1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$. Now $D_u f(3,4) = \nabla f(3,4) \cdot u$ $= (2\mathbf{i} - \mathbf{j}) \cdot (\frac{-1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}) = (2)(\frac{-1}{\sqrt{2}}) + (-1)(\frac{1}{\sqrt{2}}) = \frac{-3}{\sqrt{2}}$.