Solution to quiz 3. Use the method of Lagrange multipliers to find the point on the plane $x + 2y + z = 12$ nearest the origin.

Solution. The distance from a general point (x, y, z) to $(0, 0, 0)$ is given by $\sqrt{x^2 + y^2 + z^2}$. We will instead minimize its square $f(x, y, z) = x^2 + y^2 + z^2$. We want to minimize the distance from a point (x, y, z) on $x + 2y + z = 12$ to the origin. So our constraint on (x, y, z) is that $x + 2y + z = 12$. We let $g(x, y, z) = x + 2y + z - 12$. Now we solve $\nabla f = \lambda \nabla g$. This gives us $2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k} = \lambda(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$. So $2x = \lambda$, $2y = 2\lambda$ and $2z = \lambda$. Clearly $z = x$ and $2y = 2\lambda = 4x$ so $y = 2x$. We substitute these into $x + 2y + z = 12$ to get $x + 2(2x) + x = 12$ or $6x = 12$, $x = 2$, $z = 2$, $y = 4$. So the point is $(2, 4, 2)$.