Math 14 syllabus

Math 14, 75509, Winter 2012, MWF 9:15 - 10:20, O'Connor 206.
Professor Schaefer, O'Connor 309, 554-6899, eschaefer@scu.edu,
website: google "Ed Schaefer"

Office Hours: Tuesday 9:20 - 10:20, 2 - 3, Wednesday 8:30 - 9:00, Thursday 11 - 11:30, 3 - 4.

Midterms: Wednesday February 1 and Wednesday February 22. Final: **FRIDAY, MARCH 23, 9:10 - 12:10; NO EXCEPTIONS.** If you already bought a plane ticket and can not make that time for the final then you must change to another Math 14 this week.

Book: Thomas and Finney Santa Clara edition, Volume 2. **For homework from chapter 10, you will need to photocopy pages from someone's Volume 1.**

In this course, we will study functions of several variables.

There will be several in-class quizzes, I will drop the lowest score. If you warn me before the day of the quiz, you can take it early.

Grade: Homework 15%, Quizzes 5%, Midterms 20% each, Final 40%.

I will drop the lowest two lecture's worth of homework scores, but I EXPECT YOU TO DO ALL OF THE HOMEWORK ASSIGNMENTS. No late homework is accepted. Please turn in homework stapled and folded (so it is 4 1/4" x 11") so that your name is on top and visible. I have written some of my own problems. The answers (though not the methods of solution) to some of those are given at near the end of this syllabus. Show your work on homework, quizzes and exams. Show enough work to demonstrate to the grader that you did your own work and understand how to solve the problem.

GETTING HELP: There is a lot of help available on campus. The earliest available tutoring outside of my office hours is private tutoring offered by the Drahmann Center (DC) in Benson Center 214; the phone number is 4318. If you need a private tutor, get one early. In a few weeks, the DC will organize drop-in tutoring, 3 evenings a week. I will announce the schedule and place when I find out. The DC can also help with test anxiety and study skills.

There will soon be drop-in tutoring in O'Connor 31 (Sussman room) available during several of the hours Monday-Friday 8-5. I will announce the schedule when I receive it. The tutors are honors math majors and are usually excellent. Go into the room and ask who the tutor is. They will help you when they can. If you are really stressed out, there is the counselling center at the Benson Center 201, phone number 4172. You can visit or phone to make an appointment to speak with a counsellor.

In this course, students will learn to:

- Solve problems, including choosing and developing appropriate methods, as well as communicating mathematical ideas effectively. In this course we will emphasize the use of integrals and multivariable calculus as an important problem-solving tool.
- Use mathematical reasoning and deduction to draw valid conclusions from given information. For example, we will learn to analyze surfaces and their volumes by studying properties of their antiderivatives.
- Use and understand mathematical ideas from multiple and interconnected perspectives, including algebraic, geometric, analytical and numerical points of view. We will

combine geometric visualization with careful analytical reasoning to solve problems and connect our ideas to other disciplines.

- Understand significant mathematical ideas and results in addition to mastering efficient computational techniques. Beyond computational proficiency, we will strive to understand the meaning of our results, as well as encountering some central theorems of mathematics.

In addition to providing you with a good foundation in a fundamental area of mathematics, this course will also contribute to your skills and logical perspective that will be applicable to many other courses requiring mathematical methods and careful reasoning.

Academic Integrity: The penalty for cheating is a failing grade for the course, and the University may take further disciplinary action. All of the work that you turn in should be your own, and not that of a classmate or copied from another source. See http://www.scu.edu/studentlife/resources/academicintegrity/index.cfm for more information.

Disability accommodation policy: To request academic accommodations for a disability, students must contact the Disability Resources Office located in Benson room 216, (408) 554-4111; TTY (408) 554-5445. Students must provide documentation of a disability to Disability Resources prior to receiving accommodations.

Homework

Due Wed January 11. Read the syllabus up to this point.

Due Fri 1/13:[10.1] 2a-ℓ (but not i), 3, 4a, 7-13, 18-21, 23, 35, 40, 44. (Sol'ns in back to 11 and 13 are switched). [15.3] 1-10, 12, 13, 15, 19-22, 29, 30.

Due Fri 1/20: [16.1] 3, 8, 9, 19-21 (hint on 20: square both sides), 28. A. Sketch the graph of $h(x, y) = 3 - x^2 - y^2$. B. Level curves for the function $f(x, y)$ are shown on the pictures pdf at my website. Estimate $f(-1, 1)$ and $f(1, 1)$. From outside in are $f = 10$, $f = 20$, $f = 30$, $f = 40$. Describe the graph of $z = f(x, y)$. C. Draw level curves for the function $g(x, y) = x^2 - y^2$ for $g = -2, -1, 0, 1$ and 2. Describe what kind of graph this function has. D. Describe the level surfaces of $j(x, y, z) = x^2 + y^2 + z^2$.

Due Wed 1/25:

For problems [16.3] 7-11 and 20, find $\partial f/\partial x$, $\partial f/\partial y$, $\partial^2 f/\partial x^2$, $\partial^2 f/\partial^2 y$, $\partial^2 f/\partial x\partial y$. For [16.3] 9 also compute $\partial^2 f/\partial y\partial x$ to confirm it's the same as $\partial^2 f/\partial x\partial y$. On problem 20, if you don't recall cosh(t) then see the link at my website about hyperbolic functions. [16.3] 21, 27, 28. E. Wind-chill index I is a function of actual temperature t in centigrade and wind speed v in km/hour. So $I = f(t, v)$. The chart below gives values of I for some pairs (t, v). The numbers across the top (10, 20, 30, 40, 50) are v's, the numbers down the left (20, 16, 12, 8) are t's. The numbers inside are I's. So $f(12, 20) = 5$. Estimate $f_t(12, 20)$ and $f_v(12, 20)$ (there are several acceptable answers). What are the practical interpretations of each? I'm looking for sentences like "If the velocity is 20 and the temperature increases from 12 then the windchill ..."

$t \setminus v$	10	20	30	40	50
20	18	16	14	13	13
16	14	11	9	7	7
12	9	5	3	1	0
8	5	0	-3	-5	-6

F. The pressure in the first octant is $P(x, y, z) = x^2 + yz$. Use partial derivatives to estimate the pressure change which results from moving from $(1, 2, 3)$ to i) $(1.1, 2, 3)$ and ii) $(1, 2, 2.8)$.
G. Level curves for the function $f(x, y)$ are shown on the pictures pdf at my website. Use them to estimate $f_x(1, 1)$ and $f_y(1, 1)$.
H. i) Find the linearization to $f(x, y) = x^3y^2$ at $(2, 1)$. ii) Use it to estimate $f(2.2, .9)$. iii) Find the actual $f(2.2, .9)$ on your calculator to see how good the estimate was. iv) Find the equation of the tangent plane to $z = x^3y^2$ at $(2, 1)$.
I. Do the same as in H. with $f(x, y) = \sqrt{1 + x + 6y}$ (i.e. use $(2, 1)$ again etc). J. i) Use a linearization to approximate $\sin(0.01) + \cos(0.02) + \tan(-0.03)$. ii) Find the actual value on your calculator for comparison. K. The volume of a cone is $\frac{\pi}{3}hr^2$. You intend to build a cone with $r = 3$cm and $h = 5$cm but both measurements may be off by as much as 0.1cm. Use the principle of the differential to approximate the maximal error you might get in volume. Use your calculator to give a decimal answer. L. You intend to build a box with height 50cm, width 60cm and depth 70cm. Each measurement may be off by as much as 0.1cm. Use the principle of the differential to approximate the maximal error you might get in volume. Does the answer surprise you? Calculate (50.1)(60.1)(70.1) – (50)(60)(70) for comparison.
M. Given $f(1, 2) = 3$, $f(2, 2) = 3.5$ and $f(1, 3) = 2.8$, estimate $f(1.5, 2.5)$ and $f(2.5, 1.5)$.

Due Fri 1/27: [16.4] 1, 2, 17, 18, 20, 28 (for 28, evaluate each side and check they're the same). N. $z = f(x, y)$, $x = r\cos(\theta)$, $y = r\sin(\theta)$. Write $\partial z/\partial r$ and $\partial z/\partial\theta$ in terms of $r, \theta, \partial f/\partial x$ and $\partial f/\partial y$. Problem O. $w = f(x, y, z)$, $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$. Write $\partial w/\partial\theta$ in terms of $\partial f/\partial x$, $\partial f/\partial y$, $\partial f/\partial z$, ρ, ϕ and θ.

Due Fri 2/3: [16.6] 1, 2, 4, 9, 12, 19, 20, 27, 32, 34. [16.7]: 1a, 9a, 14a, 21, 24, 36. P. A contour map is shown (on the pictures pdf at my website) of barometric pressure (in millibars) during a hurricane. We can consider these to be level curves for the pressure function. i) Estimate the directional derivative of the pressure function at Cancun in the direction of the eye of the hurricane. ii) What are the units of the directional derivative (what per what?). Q. Estimate $\nabla f(3,4)$ for the function whose level curves are shown on the pictures pdf at my website. Think about length and direction. Answer in the form $a\mathbf{i} + b\mathbf{j}$. R. If $f(x,y) = x^2 + 4y^2$, find the gradient vector $\nabla f(2,1)$ and use it to find the tangent line to the level curve $f(x,y) = 8$ at $(2,1)$. Sketch the level curve, tangent line and gradient vector.

Due Wed 2/8: [17.1] 1, 6, 25, 28 (in section 17.1, they are looking for *local* maxima and minima). S. Find the maximal and minimal values of $f(x,y) = x^2 + y^2 - 6y$ over the triangular region bounded by $y = x$, $y = -2x$ and $y = 4$.

Due Fri 2/10: [17.3] 1, 6, 10, 21.

Due Wed 2/15: [18.1] 1, 2, 10, 11. T. You assign xy-coordinates to the inside face of a vertical dam. It's shape is a trapezoid (so all four sides are line segments). The corners are at $(0,0)$, $(10,-10)$, $(20,-10)$, $(30,0)$ (distances are measured in meters). Pressure at (x,y) is given by $\delta g D$ where $\delta = 1000$ $(\frac{\text{kilograms}}{\text{meter}^3})$ is the density of the water, $g = 9.8$ $(\frac{\text{meters}}{\text{sec}^2})$ is the magnitude of gravity and D is the depth (in meters) of the water at (x,y). So the units of pressure measure $\frac{\text{force}}{\text{area}}$. Compute the total force from pressure against the dam. (Aside: We see the units of $\delta g D$ are $\frac{\text{kilograms}}{\text{meter}^3}(\frac{\text{meters}}{\text{sec}^2})$meters $=$ kilograms$(\frac{\text{meters}}{\text{sec}^2})\frac{1}{\text{meters}^2}$ which measures mass $\cdot$ acceleration$\frac{1}{\text{area}} = \frac{\text{force}}{\text{area}}$. Note the units of Newtons are $\frac{\text{(kilograms)(meters)}}{\text{sec}^2}$ So your answer measures Newtons.) T'. Evaluate $\int_{-1}^{0}\int_{0}^{1} \frac{x}{x^2-5x+6} + y\,dydx$. You will need to use the method of partial fractions. If you don't remember how, see the link at my web site.

Due Fri 2/17: [18.1] 17, 22, 37, 39, 40.

Due Fri 2/24: [18.2] 1 ($a > 0$), 4, 8-10, 17, 25, 29 (just center of mass). Hint: $\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C$. End Hint. Feel free to use symmetry and/or high school geometry to reduce the number of double integrals. T". Find the x-coordinate of the centroid of the region bounded by $y = \sin(x)$ and the x-axis for $0 \le x \le \frac{\pi}{2}$. Show your work on the integration by parts. If you don't remember how, see the link at my website.

Due Wed 2/29: [18.3] 1, 3, 9 (that square root in the upper limit is not over the x), 14 (OK to use integral tables).
[18.4] 1, 6, 19, 24, 25, 30 (Ignore the word cylinder. The book uses the word *cylinder* to describe a surface given by an equation missing a variable.) For 19, recall the average value of f is $\frac{\int f}{\int 1}$, $\frac{\int\int f}{\int\int 1}$ or $\frac{\int\int\int f}{\int\int\int 1}$, depending on the region of integration.

Due Fri 3/2: [18.5] 4a (just centroid, answer in back: $\bar{x} = \frac{1}{4}$), 5 (just center of mass), 7 (use common sense for $\bar{x}$, $\bar{y}$, polar helps, homogeneous means constant density. So you are looking for a centroid).

Due Wed 3/7: [18.6] 1, 7, 21, 25, 29, 45 ($a = 1$), 53 (let $a = 1$; use common sense for two of them). U. Find the average value of the function z^2 over $x^2 + y^2 + z^2 \leq 9$. It's OK to use integral tables for the really awful antiderivatives here and for future homeworks.
[19.1] 1 - 9, 13, 15, 21, 29 (just center of mass, let $a = 1$). V. a) The xy-plane is horizontal. You have a fence coming up out of the xy-plane. The base of the fence is the curve $y = x^2$ from $(0, 0)$ to $(2, 4)$. The height of the fence at the point (x, y) is x. You break the curve into four pieces of equal length Δs. Let $P_1 = (x_1, y_1), P_2 = (x_2, y_2), P_3 = (x_3, y_3), P_4 = (x_4, y_4)$ be the midpoints of each of the four pieces. Approximate the area of the fence using symbols from $+$, Δs, x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4 (you may not need all of those symbols). You will not have a numerical answer. b) If we let the number of pieces of curve go to infinity, then the limit of this process gives you the exact area of the fence (OK, of one side of the fence if you are picky). Find it. You should have a numerical answer.

Due Fri 3/9: [19.2] 1, 2, 11, 19, 21, 24. On 19,21,24, let $a = 1$. On 24, do two problems, first with $\mathbf{F}_1$, then again with $\mathbf{F}_2$.

Due Wed 3/14: [19.3] 4 (let $a = 1$), 7, 11, 13, 14.
[19.5] 1, 2, 5, 6. [19.6] Just find the curl of $\mathbf{F}$ for the $\mathbf{F}$'s given in problems 1 - 3.

Due Fri 3/16: [19.7] 11, 12, 14, 19.

Some answers in the back: B. Upside down paraboloid. D. The planes normal to $\mathbf{i} + \mathbf{j} + \mathbf{k}$. E. $f_t(12, 20) \approx 1.375$. H. $12x + 16y - 32$, 8.800, 8.625. J. .98000, .97979. L. 1070 cm^3. N. $\frac{\partial z}{\partial r} = \frac{\partial f}{\partial x}\cos(\theta) + \frac{\partial f}{\partial y}\sin(\theta)$. S. Max value 8. T. $8.17 \cdot 10^6$ Newtons (which is about 918 tons). T". 1.