

NOTE: These is meant *only* to help you practice with the material we studied later in the course. The final exam will be comprehensive, so be sure to review old exams and old practice sheets.

1. The owner of a large farm with a known reservoir of gas beneath the ground sells the gas rights to a company for a guaranteed continuous money flow of $f(x) = 60 e^{-.02x}$ thousand dollars per year *forever*, where x is the number of years from now. Find the present value of this perpetual income stream, assuming an interest rate of 7% compounded continuously.

2. Given that $f(x, y) = \sqrt{x^2 + 4xy + y^2}$, find the partial derivatives $f_x(2, 0)$ and $f_{xy}(2, 0)$. Simplify your answers.

3. You have 90 hours before you take the SAT. You theorize that your total score depends on how you balance your time use between sleeping and studying. You adopt a Cobb-Douglas model and assume that your score will be

$$f(y, z) = 30 y^{\frac{7}{10}} z^{\frac{3}{10}},$$

where y is the number of hours you study and z is the number of hours you sleep. If you wanted to maximize your score, how many hours would you sleep and how many would you study?

4. A store sells x assembled computers and y do-it-yourself kits for the same machine. Their monthly profit (in dollars) is given by the model

$$P(x, y) = 480x + 400y - x^2 - xy - \frac{3}{2}y^2 - 30,000.$$

Suppose that the store can sell no more than 200 units in a month. Find the combination of sales that would maximize the profit in this situation. If there were a way to sell more than 200 units, would this lead to higher profit?

5. A. Determine the value of k so that $f(x) = k(10 - x)$ is a probability density function on the interval $[-10, 10]$. Use this value in part B.
B. The amount of time Professor Jetson will arrive before or after the start of class, is a continuous random variable with the probability density function in part A. What is the probability Professor Jetson arrives before the start of class (at $x = 0$)?
C. On the average, how many minutes, early or late, does Professor Jetson usually arrive, according to this model. Remember, the numerical answer can be negative and this would require interpretation.
6. The amount of water used (in thousands of gallons) at Terry's Classic Car Wash on a typical weekend day is thought to be a continuous random variable with the following density function:

$$f(x) = \begin{cases} \frac{1}{9}(x-1)^2 & 1 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

Verify that this is a probability density function. They have to pay a fine if they are caught using more than 3 thousand gallons in a day. If the water inspector arrives on a typical Saturday, what are the chances Terry will have to pay the fine? Also, find the expected value of this random variable. (Hint: for this last computation, integration by parts is *by far* the simplest method.)