Mark all the writing errors you can find in these proofs. Then write new proofs that are correct and clear. In one case, the argument is wrong and needs fixing; in another case, the argument is substantially correct, but hard to understand. Take as given the following fact from number theory: If a prime number is a factor of the square of \( n \), then it is also a factor of \( n \).

1. There is no rational number whose square is 2.
   \[ \text{Proof:} \] We have to show that when we square every rational number, you don’t get 2. So let \( p/q \) be a rational number, i.e. \( p \) and \( q \) are nonzero integers. Squaring that fraction and doing an obvious rearrangement, the new answer is \( p^2 = 2q^2 \), and clearly this establishes the evenness of \( p \). But then putting in a representation of \( p \) as an even number and doing the math, \( q \) was even too. But why didn’t we start with a fraction in lowest terms? If you do, then this gets us a contradiction. So we’re done.
   \[ \text{Corrected Proof:} \] Suppose there is a rational number \( p/q \) such that \( (p/q)^2 = 2 \). We can assume that the integers \( p \) and \( q \) have no common factor, meaning that the fraction is in lowest terms. Rearrangement shows \( p^2 = 2q^2 \), so \( p \) is even, and can be written as \( p = 2m \) for some integer \( m \). Substitution gives \( 4m^2 = 2q^2 \), and we see that \( q \) is even. Since \( p \) and \( q \) are both even, they have a common factor, which contradicts our assumption that they do not. This shows that there is no rational number whose square is 2.

Language critique of the original: The writer should not toggle back and forth between “we” and “you.” Instead of “the evenness of \( p \),” the author should say “that \( p \) is even.” It would be much better to talk about the fraction being in lowest terms at the beginning of the proof.

2. The graphs of the curves represented by the equations
   \[ y = x^2 - x \quad \text{and} \quad y = x/4 - 1/4 \]
   have exactly one point in common.
   \[ \text{Proof:} \] To find the point, set them equal and solve: \( x^2 - x = x/4 - 1/4 \) and \( x(x-1) = (x-1)/4 \). Cancel “common factors” and you’re looking at \( x = 1/4 \), that is the only point of intersection.
   \[ \text{Corrected:} \] The graphs of the curves represented by the equations
   \[ y = x^2 - x \quad \text{and} \quad y = x/4 - 1/4 \]
   have exactly two points in common.
   \[ \text{Proof:} \] If they have any points in common, they have the form \( (x,y) \), where the \( x \)-value gives the same corresponding \( y \) when substituted into each of the two equations. Thus \( x^2 - x = x/4 - 1/4 \) and \( x(x-1) = (x-1)/4 \). The only two solutions here are \( x = 1/4 \) and \( x = 1 \). This shows that the two graphs have exactly two points in common: \((1/4, -3/16)\) and \((1,0)\).

Language critique of original: What is meant by “them” in the first sentence? Did it make your eyes hurt to see “your looking at”? Clearly the author meant “you’re looking at.” The main error was this: When you divide an equation by a quantity, you are assuming that that quantity is nonzero; this caused the author to lose one solution.