Math 31 Farris
Calculus for Business II
Extra Problems on Lagrange Multipliers

1. A company makes *x* cheap DataFones and *y* deluxe NetPhones. Market research shows that the joint demand functions for these products are given (in dollars) by

$$p = 500 - x/10 + y/15$$
$$q = 1200 + 2x/15 - y/5,$$

where *p* and *q* are the prices of the cheap and deluxe units. The cheap units cost $100 to produce, and the deluxe models cost $400. Fixed costs are $900,000. Express the profit as a function of *x* and *y*. Find the number of each product that you should sell in order to maximize your profit, subject to the constraint that no more than 5,000 units can be sold. How would you set the prices to create this ideal situation?

2. A cruise line sells tickets to two kinds of passengers for a three-hour tour of Monterey Bay: adults and children. Children's tickets cost $50, while adults pay $80. The cost (in dollars) of taking x children and y adults is thought to be:

$$C(x, y) = 30x + 50y + .1x^2 + .2xy + .2y^2 + 300$$

Find $P(x,y)$, the profit involved in taking x children and y adults. Now suppose that you have a firm policy that there must be at least 2 adults for every child on the cruise. (Be sure to think carefully when you write this down as an inequality; many students in the past have done this incorrectly.) Also, the boat will hold at most 100 people. Find the combination of passengers that will maximize the profits.

3. Use Lagrange multipliers to find the extreme values of the function $f(x, y) = x^2 + \frac{y^2}{4}$, subject to the constraint $y = 2x^2 - 3$.

4. Now use Lagrange multipliers to find the extreme values of the function $f(x, y) = y - 2x^2$, subject to the constraint $x^2 + \frac{y^2}{4} = \frac{5}{4}$. Make a single drawing of your answers to questions 3 and 4.