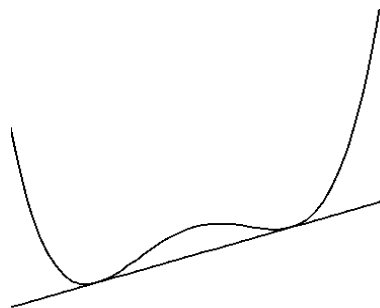


This is not a practice test. It is a set of problems to help you practice. The exam will also include some more routine problems, which you can practice using your textbook. Except when told otherwise, you may use symbolic differentiation, instead of the definition of the derivative.

1. The figure shows that the graph

$y = f(x) = x^4 - 2x^2 + x$ is tangent to a line at two different points. Given that the line has the form $y = x + b$, find those two points and determine the value of b .



2. The displacement of a particle is given by $s = 2\sqrt{t}$, for $t \geq 0$, where s is measured in meters and t in seconds. Was the average speed over the interval $[0, 16]$, greater than, equal to, or less than the instantaneous speed at $t = 4$? For the most practice, use the definition of the derivative to find the instantaneous speed.

3. Find the derivative of $f(x) = \frac{x}{x+3}$ directly from the definition, then check with the quotient rule. Use your result to find the formula for the line tangent to the graph of f at the point where $x = 0$. Graph this function by computing the following limits:

$$\lim_{x \rightarrow \infty} f(x) \quad \lim_{x \rightarrow -\infty} f(x) \quad \lim_{x \rightarrow -3^+} f(x) \quad \lim_{x \rightarrow -3^-} f(x)$$

4. Let's think about the function $f(x) = -1 - \frac{1}{x}$ and also the composite function $g(x) = f(f(x))$. Find the derivatives $f'(x)$ and $g'(x)$. (You will need a simple formula for g . You may use symbolic differentiation, or use the definition for extra practice.) For more fun, find the derivative of $h(x) = f(f(f(x)))$.

5. Be a function detective: The mystery function is defined by the formula

$$f(x) = \begin{cases} ax^2 + bx + 1, & -2 \leq x < 0 \\ cx + d, & 0 \leq x < 2 \\ qx + r, & 2 \leq x \leq 3. \end{cases}$$

Use these clues about f to determine a , b , c , d , q , and r : The tangent line is horizontal when $x = -1$,

$$f(-1) = \lim_{x \rightarrow 0} x \cos(x), \quad f'(1) = \lim_{x \rightarrow 0} \frac{\sin(2x)}{4x}, \quad \lim_{x \rightarrow 2^-} f(x) = 1, \quad f(2) = 1, \quad \text{and} \quad f(3) = \lim_{x \rightarrow \infty} \frac{47}{x}.$$

Then answer: At what points is f continuous?