1.) a) \( P_{3}^{10} = 10 \cdot 9 \cdot 8 = 720 \)

b) \( C_{2}^{1} \cdot C_{1}^{3}! \) or \( (\frac{10}{2}) \cdot \frac{9 \cdot 8}{2!} = 9 \cdot 8 \cdot 3 = 216 \)

c) \( P_{2}^{3} = 9 \cdot 8 = 72 \)

d) \( C_{2}^{1} \cdot C_{4}^{3}! \) or \( (\frac{10}{2}) \cdot (\frac{6}{4}) \cdot 3! = 6 \cdot 5 \cdot 4 \cdot 3 = 360 \)

2.)

\[ \frac{1}{3} \frac{2}{3} \frac{1}{3} \frac{2}{3} \frac{1}{3} \]

a) Sample Point method: \( \frac{2 \cdot 3}{15} = \frac{2}{5} \)

Event Composition method: RW or WR

\[ P(R\{RW\} W) = \frac{2}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{2}{5} = \frac{2}{5} \]

b) \( P(R\{RW\}) = \frac{1}{6} \cdot \frac{3}{5} = \frac{1}{10} \)

c) Sum up all branches leading to second ball is white:

\[ P(\text{second white}) = \frac{2}{6} \cdot \frac{3}{5} + \frac{3}{6} \cdot \frac{3}{5} + \frac{1}{6} \cdot \frac{3}{5} = \frac{1}{2} \]

3.) a) \( P(\text{1st card is ace of spades}) = \frac{1}{52} \)

Likewise, \( P(\text{2nd card is ace of spades}) = \frac{1}{52} \)

So \( P(\text{one of 13 cards is ace of spades}) = P(\text{1st card is ace of spades}) + P(\text{2nd card is ace of spades}) + \cdots + P(\text{13th card is ace of spades}) \)

\[ = \frac{1}{52} + \frac{1}{52} + \cdots + \frac{1}{52} = \frac{13}{52} \]

b) Let \( B \) = random one of 13 cards is not the ace of spades

\( A = \) ace of spades dealt among the 13 cards \( \Rightarrow P(A) = \frac{13}{52} \)

\( A' = \) ace of spades NOT dealt among the 13 cards \( \Rightarrow P(A') = \frac{39}{52} \)

So \( P(A'|B) = \frac{P(B|A') \cdot P(A')}{P(B|A') \cdot P(A') + P(B|A) \cdot P(A)} \)

\[ = \frac{1 \cdot \frac{39}{52}}{\frac{39}{52} \cdot \frac{13}{13} + \frac{39}{52} \cdot \frac{13}{52}} = \frac{\frac{39}{52}}{\frac{51}{52}} = \frac{13}{17} \]

c) Suppose repeated 10 times

Let \( C = \) ace of spades is not seen 10 times

\[ \frac{P(A|C)}{P(A)} = \frac{P(A|C)}{P(A|C) + P(A|C')} \]

\[ \approx 0.1302 \]
4.) a) \( P(Y=0) + P(Y=1) \) where \( Y = \# \) of defective items

\[
P(Y=0) + P(Y=1) = \binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{1}(0.1)^1(0.9)^{9} \approx 0.7361
\]

b) Poisson approximation: \( \lambda = (10)(0.1) = 1 \)

\[
P(Y=0) + P(Y=1) = \frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} = e^{-1} + e^{-1} \approx 0.7358
\]

5.) \( Y = \# \) of total of first 6 tossed

\( p = \frac{1}{6} \) \( \therefore \lambda = \frac{5}{6} \)

\[
P(Y=4 \mid Y \geq 2) = \frac{P(Y=4 \land Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y=4)}{P(Y \geq 2)} = \frac{(\frac{5}{6})^3(\frac{1}{6})}{(\frac{5}{6})^3(\frac{1}{6})} = \frac{(\frac{5}{6})^3(\frac{1}{6})}{(\frac{5}{6})^1} \approx 0.1157
\]

6.) Note: since all sample points in the event of 4 defective and 6 non-defective are equally likely, this problem boils down to

\[
\frac{(\frac{5}{6})^3(\frac{1}{6})}{(\frac{5}{6})^3(\frac{1}{6})} = 0.476
\]

Long way:

Let \( Y_1 = \# \) of defectives from line 1

\( Y_2 = \# \) of defectives from line 2

Both are binomial with \( n=5 \) and defective prob = \( p \).

Then \( Y_1 + Y_2 \) is binomial with \( n=10 \) and defective prob = \( p \).

\[
P(Y_1 = 2 \mid Y_1 + Y_2 = 4) = \frac{P(Y_1 = 2)P(Y_2 = 2)}{P(Y_1 + Y_2 = 4)} = \frac{(\frac{5}{6})^2(\frac{1}{6})^3(\frac{5}{6})^2(\frac{1}{6})^3}{(\frac{10}{6})^4} \approx \frac{(\frac{5}{6})(\frac{1}{6})}{(\frac{10}{6})(\frac{1}{6})}
\]

7.) Let \( A \subset B \)

Then \( P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \)

Then \( A \) & \( B \) are only independent if \( P(B) = 1 \), so that \( P(A \mid B) = P(A) \)