Math 123 Midterm 1: You may use a cheat sheet, calculator, and the Z-, T-, and χ² tables.

1. A quality control engineer is checking the amount of ibuprofen in a certain company’s painkiller drug. In a given day, the pills produced that day are considered acceptable if the average amount of ibuprofen is at least 190 mg.
   a. He selects a random sample of 40 pills and finds that the sample mean is 200 mg with a sample standard deviation of 50 mg. How confident are you that the batch of pills produced today is acceptable?
   b. The engineer is unhappy with his findings in part (a.) and wants to be 95% confident that the error of estimation is less than 3.0. He suspects the population standard deviation is 50 mg. How many pills should he sample to achieve this?

2. A researcher decides to conduct a poll at a university which asks college students how many drinks they typically drink in a given week. The researcher randomly picks 4 college students and gets the responses of 4 drinks, 7 drinks, 10 drinks, and 15 drinks per week.
   a. Find a 95% confidence interval for the mean number of drinks that college students at this university typically drink in a week. What assumptions are necessary?
   b. Find a 95% confidence interval for σ², the variance of the number of drinks that college students at this university typically drink in a week. What assumptions are necessary?

3. Let Y₁, Y₂, ..., Yₙ be a random sample of observations from a Bernoulli distribution with probability function
   \[ p(Y_i) = p^{y_i}(1 - p)^{1-y_i} \] for \( y_i = 0, 1 \) and \( i = 1, 2, ..., n \).
   a. Show that \( \sum Y_i \) is sufficient for \( p \).
   b. Find an MVUE for \( p \) (Hint: this is also the MLE of \( p \), but you do not need to prove this).
   c. Find an MVUE of the population variance \( p(1 - p) \).

4. Let Y₁, Y₂, ..., Yₙ be a random sample of observations from a normal distribution with density
   \[ f(Y_i) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(Y_i - \mu)^2}{2\sigma^2}} \] for \( -\infty < Y_i < \infty \) and \( i = 1, 2, ..., n \).
   a. Find the method of moments estimators to estimate the parameters \( \mu \) and \( \sigma^2 \).
   b. Show that the estimator you found for \( \sigma^2 \) in part (a.) is biased.
   c. Find an unbiased estimator of \( \sigma^2 \) by an appropriate scaling of your biased estimator.

5. Let Y₁, Y₂, ..., Yₙ be a random sample of observations from a geometric distribution with probability function
   \[ p(Y_i) = p(1 - p)^{y_i-1} \] for \( y_i = 1, 2, 3, ... \) and \( i = 1, 2, ..., n \).
   a. Find the MLE of \( p \).
   b. Find the MLE of \( 1/p \), the population mean.
   c. Show that the estimators you found in parts (a.) and (b.) are both consistent estimators.

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