1.a.) \( H_0: \theta = 0.14 \)
\( H_a: \theta > 0.14 \)
Assume \( H_0: \theta = 0.14 \)
Test statistic \( Z = \frac{\hat{\theta} - \theta_0}{\sqrt{\frac{\theta_0(1-\theta_0)}{n}}} = \frac{0.18 - 0.14}{\sqrt{\frac{0.14(0.86)}{66}}/897} = 3.45 \)
For \( \alpha = 0.01 \), \( z_{\alpha} = 2.33 \)  
\( \text{So since } 3.45 > 2.33, \text{ we can reject the null hypothesis and accept the alternative hypothesis that the debate increased the percentage of South Carolina republicans who will vote for Marco Rubio.} \)

b.) \( p\text{-value} = P(Z > 3.45) = P(Z > 3.5) = 0.000233 \)
So for \( \alpha \geq 0.000233 \), we can reject \( H_0 \).

2.) a.) \( F = \frac{S_1^2}{S_2^2} = \frac{\chi^2_{n_1-1}/(n_1-1)}{\chi^2_{n_2-1}/(n_2-1)} = \frac{(n_1-1)S_1^2/\sigma_1^2}{(n_2-1)S_2^2/\sigma_2^2} = \frac{S_1^2}{S_2^2} \)
Assume \( H_0: \sigma_1^2 = \sigma_2^2 \rightarrow \frac{S_1^2}{S_2^2} \)  
Thus \( \frac{S_1^2}{S_2^2} \) has an \( F \) dist with \( n_1-1 \) numerator df & \( n_2-1 \) denominator df.

b.) \( H_a: \sigma_1^2 > \sigma_2^2 \)
So let \( S_1^2 = 7 \), \( S_2^2 = 5 \)
\( \frac{S_1^2}{S_2^2} = \frac{7}{5} = 1.4 \)  
For \( \alpha = 0.1 \), \( F_{\alpha} = 2.51 \)
Since 1.4 < 2.51, we cannot reject the null hypothesis.
That the variance in sugar is the same for both companies.

3.) Binomial Distribution
\( p(y) = \binom{n}{y} p^y (1-p)^{n-y} \)  
Let \( Y_i \) = number of goals scored by \( i \)th player

\( H_0: Y_i \) has a binomial distribution
\( H_a: Y_i \) has some other distribution
\( \theta \) for Binomial has MLE \( \bar{Y} = \frac{\sum Y_i}{n} \)

\( p = \frac{0.0 + 1.2 + 2.9 + 3.1}{66} = \frac{5.3}{66} \approx 0.08 \)
The probability for $0$ goals is $p(0) = \binom{3}{0} (0.8)^0 (0.2)^3 = 0.008$

The probability for $1$ goal is $p(1) = \binom{3}{1} (0.8)^1 (0.2)^2 = 0.096$

The probability for $2$ goals is $p(2) = \binom{3}{2} (0.8)^2 (0.2)^1 = 0.384$

The probability for $3$ goals is $p(3) = \binom{3}{3} (0.8)^3 (0.2)^0 = 0.512$

So $n_1 = 11$ & $n_2 = 11$ where $n_1+n_2=22=n$

**Combined**

$E(n_1) = n p_1 = (22)(0.488) = 10.736$

$E(n_2) = n p_2 = (22)(0.512) = 11.264$

$X^2 = \frac{(10.736-11)^2}{10.736} + \frac{(11.264-11)^2}{11.264} = 0.0127$

$X^2$ with $k-1-1$ df

But $k=2$ here, so just subtract $1 = 2-1 = 1$ df

So $X^2$ with $1$ df for $\alpha = 0.05$ is $3.84$

Since $0.0127 < 3.84$, we **cannot reject** the null hypothesis that $Y_i$ has a binomial distribution.

**4. a)**

$X^2 = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} = 0 \implies n_i = np_i$

Here $p_i = \frac{1}{k}$; so $n_i = \frac{n}{k}$ for $i = 1, \ldots, k$

**b)**

$X^2 = \sum_{i=1}^{k} \frac{(cn_i - cnpi)^2}{cnp_i} = \sum_{i=1}^{k} \frac{c^2(n_i - np_i)^2}{c(np_i)} = c \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i}$

So The value of $X^2$ is scaled by $c$, where $c$ is a positive integer.

Thus, we are more likely to reject $H_0 : p_i = \frac{1}{k}$

**5.**

Let $Y_i = \#$ trials in cell $i$.

Combine all cells, except $i$, into a single large cell.

Then every cell is in cell $i$ or not in cell $i$, with probabilities $p_i$ or $1-p_i$.

So $Y_i$ has a binomial marginal probability distribution.

Thus $E(Y_i) = np_i$ & $V(Y_i) = np_i(1-p_i)$.