1.) a) \( 0.5166 = 0.49 - z_\alpha \sqrt{\frac{0.049(0.51)}{427}} \)  \\
\(-z_\alpha = \frac{0.0266}{\sqrt{\frac{0.049(0.51)}{427}}} \Rightarrow -z_\alpha = 1.1 \)  \\
\( P(Z > -z_\alpha) = P(Z > 1.1) = 0.1357 \)  \\
\( \Rightarrow \alpha = 0.8643 \Rightarrow 1 - \alpha = 0.1357 \)

So only 13.57% chance that Donald Trump will clinch the nomination.

b.) We didn't account for the fact that some Republican candidates might drop out between now and the remaining primaries and that some states are winner-take-all.

2.) a.) \( L(y_1, y_n | \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_1 - \mu)^2}{2\sigma^2}} \cdot \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_2 - \mu)^2}{2\sigma^2}} \cdots \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_n - \mu)^2}{2\sigma^2}} \)

\( = \left(\frac{1}{\sigma \sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma^2} \left( \sum_{i=1}^{n} (y_i - \mu)^2 \right)} \)

Thus, by factorization thm, \( \sum_{i=1}^{n} (y_i - \mu)^2 \) is sufficient for \( \sigma^2 \)

b.) We have \( E \left( \sum_{i=1}^{n} (y_i - \mu)^2 \right) = \sum_{i=1}^{n} E((y_i - \mu)^2) = \sum_{i=1}^{n} \sigma^2 = n\sigma^2 \)

Thus if \( U = \sum_{i=1}^{n} (y_i - \mu)^2 \), then \( E \left( \frac{U}{n} \right) = \frac{1}{n} E(U) = \frac{1}{n} n\sigma^2 = \sigma^2 \)

Thus by Rao-Blackwell Thm, an MVUE for \( \sigma^2 \) is \( \frac{1}{n} \sum_{i=1}^{n} (y_i - \mu)^2 \)

c.) Recall \( Y_i \), \( i \) \( \epsilon \) \( \{1, \ldots, n\} \) are normally distributed.

Thus \( \sum_{i=1}^{n} (Y_i - \mu)^2 \) has a \( \chi^2 \) distribution with \( n \) degrees of freedom.

Thus \( \frac{n}{\sigma^2} \left( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \right) \) has a \( \chi^2 \) distribution with \( n \) degrees of freedom.

Recall \( V(\chi^2_n) = 2n \)

Thus \( V \left( \frac{n}{\sigma^2} \left( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \right) \right) = V(\chi^2_n) = 2n \)

\( \Rightarrow \frac{n^2}{\sigma^4} V \left( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \right) = 2n \Rightarrow V \left( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \right) = \frac{2\sigma^4}{n} \)

So since \( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \) is unbiased by part b.) & \( \lim_{n \to \infty} \left( \frac{n}{\sigma^2} \left( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \right) \right) = \lim_{n \to \infty} \frac{2\sigma^4}{n} = 0 \)

Then \( \frac{1}{n} \sum_{i=1}^{n} (Y_i - \mu)^2 \) is consistent (assuming \( \sigma^2 \) is finite).
3. \( H_0: \mu_1 - \mu_2 = 0 \)

\( H_a: \mu_1 - \mu_2 \neq 0 \)

\[
Z = \frac{\bar{Y}_1 - \bar{Y}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{56,941 - 56,530}{\sqrt{\frac{(65)^2}{136} + \frac{(6430)^2}{152}}} = \frac{408}{765.898} = 0.5366
\]

For \( \alpha = 0.05 \), \( \frac{\alpha}{2} = 0.025 \) \( \Rightarrow Z_{\alpha/2} = 1.96 \)

So since 0.5366 < 1.96, we cannot reject the null hypothesis that there is a difference in salaries in the two years.

b) \( p \)-value:

\[
P(Z > 0.5366) = 2 \cdot (0.2946) = 0.5892
\]

So for \( \alpha = 0.5892 \), we can reject the null hypothesis.

4. a) \[
\begin{bmatrix}
1 & A \\
2 & B
\end{bmatrix}
\]

Columns swapped: \[
\begin{bmatrix}
A & B \\
B & A
\end{bmatrix}
\]

Row probabilities: \( \frac{a+b}{n} \) & \( \frac{c+d}{n} \)

Column probabilities: \( \frac{a+c}{n} \) & \( \frac{b+d}{n} \)

Since row & column probabilities remain unchanged, then \( E(n_{ij}) \) is unchanged for every \( n_{ij} \) (with respect to the new labeling), so \( X^2 \) stays the same.

b) Same is true if rows are swapped.

c) Since we can swap rows & columns without changing \( X^2 \), then taking the transpose: \[
\begin{bmatrix}
1 & A \\
2 & B
\end{bmatrix}
\]
results in the same value for \( X^2 \).

5. \[
\begin{bmatrix}
\beta_0 \\
\beta_1
\end{bmatrix}
= \left( \sum_{i=1}^{n} x^2_i \right)^{-1} \begin{bmatrix}
\sum_{i=1}^{n} x_i y_i \\
\sum_{i=1}^{n} x_i^2
\end{bmatrix}
\]

where \( n = 7 \), \( \sum_{i=1}^{n} x_i = 21 \), \( \sum_{i=1}^{n} x_i^2 = 91 \)

\[
\begin{bmatrix}
\beta_0 \\
\beta_1
\end{bmatrix}
= \left[ \begin{bmatrix} 21 & 91 \end{bmatrix}^{-1} \begin{bmatrix} 4261 \\
14183
\end{bmatrix} \right] = \frac{1}{7(91) - 21^2} \begin{bmatrix} 91 - 21 \\
-21 & 7
\end{bmatrix} \begin{bmatrix} 4261 \\
14183
\end{bmatrix}
= \frac{1}{146} \begin{bmatrix} 899.08 \\
98
\end{bmatrix}
= \begin{bmatrix} 4.587 \\
0.5
\end{bmatrix}
\]

So \( \hat{\beta}_0 = 4.587 \) & \( \hat{\beta}_1 = 0.5 \)

\( g = 4.587 + 0.5 \times \)

Since the slope is positive, this suggests an increase in median prices over time. The expected annual increase is $50,000...

Which means, in a related story, that I'm never going to be able to afford buying a home in Santa Clara County. 😞