Math 36 Sample Final

1. Find the following integrals:
   a. \[ \int \frac{\cos x}{(1-\sin x)(2-\sin x)} \, dx \]
   b. \[ \int e^x \sin x \, dx \]

2. Use the survival-renewal function \( s(T)N_0 + \int_0^T s(T-t)r(t)dt \) for the following:
   a. I initially invest $1000 in a savings account and put in $1000 per year. The money in this account is compounded continuously at a rate of 5% per year. How much will I have in the account in 20 years?
   b. The term \( s(T)N_0 = e^{(0.05)(20)} \cdot 1000 \) is a solution to which differential equation at \( t = 20 \) years? What is the initial condition \( N(0) \)?

3. Draw the graph of the solution to the logistic equation \( \frac{dN}{dt} = rN \left( 1 - \frac{N}{K} \right) \)
   when \( r > 0 \) and \( K > 0 \) and find the solution \( N(t) \) when \( N(0) = N_0 < K \).

4. Draw the bifurcation diagram for \( \frac{dy}{dt} = y - ay^3 \) assuming \( y \geq 0 \) and determine if the equilibria are stable, unstable, or semistable.

5. Use the comparison test to determine if \( \int_1^\infty \frac{1}{1+e^x} \, dx \) is convergent or divergent.

6. For the exponential PDF \( f(x) = ce^{-cx} \) with \( c > 0 \) for \( x \geq 0 \) and zero otherwise:
   a. Find the mean
   b. Find the standard deviation.

7. If \( X \) has a normal distribution with mean \( \mu \) and standard deviation \( \sigma \), find the probability that \( X \) lies within one standard deviation from the mean, i.e. \( P(\mu - \sigma \leq X \leq \mu + \sigma) \).