More problems from Chapter 10:

A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets U and V such that every edge connects a vertex in U to one in V. Equivalently, a bipartite graph is a graph that does not contain any odd-length cycles.

1. Show that a simple graph is bipartite if and only if it is 2-colorable.

2. Prove that a simple graph with p vertices and q edges is complete (has all possible edges) if and only if \( q = p(p-1)/2 \).

3. Prove that, if T is a tree, then for all edges e, T-e (i.e. T with the edge e removed) has exactly two components.