Math 51 Sample Midterm 2

1. For each of the following, determine if it is true or false. Justify your answers.
   a.  $A \cup (B \cap C) = (A \cup B) \cap C$.
   b.  $\emptyset \subseteq \emptyset$.
   c.  Let $A = \{\emptyset, 1, \{2\}\}$. Then $\{\emptyset, \{2\}\} \subseteq \mathcal{P}(A)$.

2. Let $A, B,$ and $C$ be sets. Prove if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

3. For the function $f: (1, \infty) \to (0, \infty)$ given by $f(x) = \frac{1}{x+1}$,
   a.  Is $f$ 1-1? Either prove it is 1-1 or explain why it is not.
   b.  Is $f$ onto? Either prove it is onto or explain why it is not.

4. Find the following sums.
   a.  $\sum_{i=1}^{4} 3 \cdot 2^{-i+1}$.
   b.  $\sum_{i=2}^{6} 2^i + i$.

5. Prove that the set $\{x \in \mathbb{Z} : x < -12\}$ is denumerable.

6. Prove that the set $(1, \infty)$ has cardinality $\mathfrak{c}$. 