24. a.) Domain: \( \exists \) students in your class \( P(x) \) where \( P(x) = x \) has a cell phone
   \( \forall \) all people \( (\forall x)(P(x) \rightarrow Q(x)) \) where \( P(x) = x \) is a student in your class
   \( Q(x) = x \) has a cell phone

   b.) Domain: \( \exists \) students in your class \( P(x) \) where \( P(x) = x \) has seen a foreign movie
   \( \forall \) all people \( (\exists x)(P(x) \land Q(x)) \) where \( P(x) = x \) is a student in your class
   \( Q(x) = x \) has seen a foreign movie

   c.) Domain: \( \exists \) students in your class \( P(x) \) where \( P(x) = x \) can not swim
   \( \forall \) all people \( (\exists x)(P(x) \land Q(x)) \) where \( P(x) = x \) is a student in your class
   \( Q(x) = x \) can not swim

   d.) Domain: \( \exists \) students in your class \( P(x) \) where \( P(x) = x \) can solve quadratic eqs
   \( \forall \) all people \( (\exists x)(P(x) \rightarrow Q(x)) \) where \( P(x) = x \) is a student in your class
   \( Q(x) = x \) can solve quadratic eqs

   e.) Domain: \( \exists \) students in your class \( P(x) \) where \( P(x) = x \) does not want to bench
   \( \forall \) all people \( (\exists x)(P(x) \land Q(x)) \) where \( P(x) = x \) is a student in your class
   \( Q(x) = x \) does not want to be rich

32. a.) All dogs have fleas.
   Domain: \( \forall \) all dogs \( P(x) = x \) has fleas
   Original: \( (\forall x)(P(x)) \) Negation: \( (\exists x) \sim P(x) \) Some dogs do not have fleas.

   b.) There is a horse that can add.
   Domain: \( \exists \) horses \( P(x) = x \) can add
   Original: \( (\exists x)(P(x)) \) Negation: \( (\forall x) \sim P(x) \) No horses can add.

   c.) Every koala can climb.
   Domain: \( \forall \) koalas \( P(x) = x \) can climb
   Original: \( (\forall x)(P(x)) \) Negation: \( (\exists x) \sim P(x) \) Some koalas can not climb.

   d.) No monkey can speak French.
   Domain: \( \forall \) monkeys \( P(x) = x \) can speak French
   Original: \( (\exists x) \sim P(x) \) Negation: \( (\forall x)(P(x)) \) Some monkeys can speak French.

   e.) There exists a pig that can swim and catch fish.
   Domain: \( \exists \) pigs \( P(x) = x \) can swim \( Q(x) = x \) can catch fish
   Original: \( (\exists x)(P(x) \land Q(x)) \) Negation: \( (\forall x)(P(x) \rightarrow \sim Q(x)) \) For every pig, if it can swim, then it can't catch fish. Alternatively, Negation: \( (\forall x)(P(x) \lor Q(x)) \) Every pig either can't swim or can't catch fish.

35. a.) \( \forall x(x^2 \geq x) \) No counterexample
   b.) \( \forall x(x > 0 \lor x < 0) \) \( x = 0 \)
   c.) \( \forall x(x = 1) \) \( x = 2 \) or any number besides 1.
1. a.) \( \forall x \forall y (x < y) \) For every real number \( x \) there exists a real number \( y \) such that \( x \) is less than \( y \).

b.) \( \forall x \forall y (((x \geq 0) \land (y \geq 0)) \Rightarrow (x \cdot y \geq 0)) \) For every real number \( x \) and real number \( y \), if \( x \) and \( y \) are both nonnegative, then their product is nonnegative.

c.) \( \forall x \forall y \exists z (x \cdot y = z) \) For every real number \( x \) and real number \( y \), there exists a real number \( z \) such that the product of \( x \) and \( y \) equals \( z \).

9. b.) Everybody loves somebody \( (\forall x)(\exists y) \) \( L(x,y) \)

c.) There is somebody whom everybody loves \( (\exists y)(\forall x) \) \( L(x,y) \)

d.) Nobody loves everybody \( (\forall x)(\exists y) \) \( \neg L(x,y) \) or \( \neg (\exists y)(\forall x) \) \( L(x,y) \)

e.) There is somebody whom no one loves, \( (\exists x)(\forall y) \) \( \neg L(x,y) \) or \( \neg (\forall x)(\exists y) \) \( L(x,y) \)

9. There is exactly one person whom everybody loves, \( (\exists x)(\forall y) \) \( L(y,x) \)

h.) Everybody loves himself or herself, \( (\forall x) \) \( L(x,x) \)

j.) There is someone who loves no one besides himself or herself, \( (\exists x)(\forall y)(\neg L(x,y) \Rightarrow \neg L(x,y)) \)

38. a.) Every student in this class likes mathematics

Domain: \{Students in this class\} \( P(x) = x \) likes mathematics

Original: \( (\forall x)P(x) \) Negation: \( (\exists x)\neg P(x) \) Some student in this class does not like mathematics

b.) There is a student in this class who has never seen a computer.

Domain: \{Students in this class\} \( P(x) = x \) has never seen a computer

Original: \( (\exists x)P(x) \) Negation: \( (\forall x)\neg P(x) \) Every student in this class has seen a computer.

c.) There is a student in this class who has taken every mathematics course offered at this school.

x-Domain: \{Students in this class\} \( P(x,y) = x \) has taken \( y \) mathematics course

Original: \( (\exists x)(\forall y)P(x,y) \)

Negation: \( (\forall x)(\exists y)\neg P(x,y) \)

Every student has not taken some mathematics course at this school.

d.) There is a student in this class who has been in at least one room of every building on campus.

x-Domain: \{Students in this class\} \( P(x,y,z) = x \) has been in \( y \) of \( z \).

Original: \( (\exists x)(\forall y)(\exists z)P(x,y,z) \)

Negation: \( (\forall x)(\exists y)(\forall z)\neg P(x,y,z) \)

Every student has not been in every room of some building on campus.
9. b.) \( P = \text{I eat spicy foods}, \quad Q = \text{I have strange dreams}, \quad R = \text{there is thunder while I sleep} \)

Premises: \( P \Rightarrow Q, \quad R \Rightarrow Q, \quad \neg Q \)

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
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</table>
| 1.   | \( \neg P \Rightarrow Q \)  
| 2.   | \( R \Rightarrow Q \)       | Premise |
| 3.   | \( \neg Q \)                  | Premise |
| 4.   | \( \neg Q \)                  | Modus Tollens using (2) & (3) |
| 5.   | \( \neg Q \)                  | Modus Tollens using (1) & (3) |
| 6.   | \( \neg Q \land \neg Q \)    | Conjunction using (4) & (5) |

Conclusion: \( \text{I did not eat spicy foods and it did not thunder} \)

c.) \( P = \text{I am clever}, \quad Q = \text{I am lucky}, \quad R = \text{I will win the lottery} \)

Premises: \( P \land Q, \quad \neg Q, \quad Q \Rightarrow R \)

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<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>( P \lor Q )</td>
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<tr>
<td>2.</td>
<td>( \neg Q )</td>
</tr>
<tr>
<td>3.</td>
<td>( Q \Rightarrow R )</td>
</tr>
<tr>
<td>4.</td>
<td>( P )</td>
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</tbody>
</table>

Conclusion: \( \text{I am clever or \"it\"} \)

d.) \( P = \text{Something is good for corporations}, \quad Q = \text{Something is good for the U.S.}, \quad R = \text{Something is good for you}, \quad S = \text{buy lots of stuff} \)

Premises: \( P \Rightarrow Q, \quad Q \Rightarrow R, \quad S \Rightarrow P \)

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<td>1.</td>
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<td>2.</td>
<td>( Q \Rightarrow R )</td>
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<tr>
<td>3.</td>
<td>( S \Rightarrow P )</td>
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<td>4.</td>
<td>( S \Rightarrow R )</td>
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<tr>
<td>5.</td>
<td>( S \Rightarrow R )</td>
</tr>
<tr>
<td>6.</td>
<td>( S \Rightarrow R )</td>
</tr>
</tbody>
</table>

Conclusion: \( \text{If I buy lots of stuff, then it is good for the U.S. and good for you} \)

1. Let \( x, y \in \mathbb{Z} \). Assume that \( x \) and \( y \) are odd. Then \( x = 2k+1 \) and \( y = 2l+1 \) for some \( k, l \in \mathbb{Z} \). Then \( xy = 2k+1 \cdot 2l+1 = 2k+2l+2 = 2(k+l+1) \). Since \( k+l+1 \in \mathbb{Z} \), then \( xy \) is even.

2. Let \( x, y \in \mathbb{Z} \). Assume that \( x \) and \( y \) are even. Then \( x = 2k \) and \( y = 2l \) for some \( k, l \in \mathbb{Z} \). Then \( xy = 2k \cdot 2l = 2(k+l) \). Since \( k+l \in \mathbb{Z} \), then \( xy \) is even.

6. Let \( x, y \in \mathbb{Z} \). Assume that \( x \) and \( y \) are odd. Then \( x = 2k+1 \) and \( y = 2l+1 \) for some \( k, l \in \mathbb{Z} \). Then \( xy = (2k+1)(2l+1) = 4kl+2k+2l+1 = 2(2kl+k+l)+1 \). Since \( 2kl+k+l \in \mathbb{Z} \), then \( xy \) is odd.
9. Assume that there exists a rational number \( x \) and an irrational number \( y \) whose sum is a rational number \( z \). So \( x + y = z \) where \( x = \frac{a}{b} \) and \( z = \frac{c}{d} \) for some \( a, b, c, d \in \mathbb{Z} \) with \( b, d \neq 0 \). Then \( y = \frac{c}{d} - \frac{a}{b} = \frac{bc - ad}{bd} \). Since \( bc - ad, bd \in \mathbb{Z} \) with \( bd \neq 0 \), then \( y \) is rational, a contradiction.

10. Assume that \( x, y \in \mathbb{Q} \). Then \( x = \frac{a}{b} \) and \( y = \frac{c}{d} \) for some \( a, b, c, d \in \mathbb{Z} \) with \( b, d \neq 0 \). Then \( xy = \frac{ac}{bd} \). Since \( ac, bd \in \mathbb{Z} \) with \( bd \neq 0 \), then \( xy \in \mathbb{Q} \).

15. Assume that \( x < 1 \) and \( y < 1 \) where \( x \) and \( y \) are real numbers. Then \( x + y < 1 + 1 = 2 \), so \( x + y < 2 \). Thus \( x + y < 2 \Rightarrow x < 1 \) or \( y < 1 \).

*Note: the book says "positive integer", but we don't actually need positivity.

27. \( \Rightarrow \) Let \( n \) be an integer. Assume \( n \) is odd. Then \( n = 2k + 1 \) for some \( k \in \mathbb{Z} \). Then \( S_{n+6} = 5(2k+1) + 6 = 10k + 5 + 6 = 10k + 11 = 10(k + 1) + 1 = 2(5k + 5) + 1 \). Since \( 5k + 5 \in \mathbb{Z} \), then \( S_{n+6} \) is odd.

\( \Leftarrow \) Let \( n \) be an integer. Assume \( n \) is even. Then \( n = 2k \) for some \( k \in \mathbb{Z} \). Then \( S_{n+6} = 5(2k) + 6 = 10k + 6 = 2(5k + 3) \). Since \( 5k + 3 \in \mathbb{Z} \), then \( S_{n+6} \) is even.