MATH 163/COEN 179 Assignment 3

23-1a
a. Consider a square-shaped graph with weighted edges 2, 3, 4, 5 (in clockwise order) and one diagonal edge of weight 1, so that the 1, 2, and 3 edges form a cycle. The minimal spanning tree has the 1, 2, and 4 edges, and the second best has either the 1, 2, and 5 edges or the 1, 3, and 4 edges. The argument that the minimal spanning tree is unique for distinct edges was given in class.

b. Suppose not. Then it is impossible to improve any second-best spanning tree $T$ by deleting one edge in the second-best tree and replacing it with any other edge which lies on some cycle with the deleted edge. Thus, any edge not found in $T$ has a larger weight than any edge in $T$ that lies on a cycle that adding the first edge would complete. Consequently, Kruskal’s algorithm would never add such an edge to its tree. Thus Kruskal’s must produce $T$, a contradiction.

4.
DFS: ABDCEFLMKNIHJ
BFS: ABCDFHNEJKGILM
The graph is connected, but not biconnected.
Articulation nodes: F, G, N
Backvalues: B, C, D, E, F, H, K, N all have backvalue 1 (node A). J has backvalue 2 (B). G, L, M, have backvalue 7 (G). I has backvalue 11 (N).

5. Vertices are popped when there are no edges left to visit new vertices. If the first vertex popped has positive outdegree, then there is an edge from it which leads to an old vertex. This would produce a cycle. If the graph is acyclic, then the first vertex popped must have outdegree 0. All edges into it would come from something popped later. Removing this vertex and the edges pointing in from consideration, and reiterating, we see that all edges in the graph will point from a tail which was popped later than the tip to which it points.

6. Consider the cut formed by separating the source and the vertices associated to the A and AB supplies and donors from the sink and the vertices associated to the B and O supplies and donors. The capacity of this cut is $14+25+19+30=88$, meaning that no more than 88 patients may be served, so not all patients will get their unit of blood. This turns out to be the minimum cut, so there is a legal flow that will get a unit of blood to 88 of the 89 patients. (In layman’s terms, after the type A and AB patients have been treated, there are at most $14+25 = 39$ units of type B or type O blood. As there are $18+22 = 40$ patients with type B or type O blood and they can only receive type B or O blood, one of them must be left out.