1. Find a Möbius transformation $T$ with the property that $T(0) = 1$, $T(1) = \infty$ and $T(i) = i$. What is the image of the right half-plane $\mathcal{R} = \{z = x + iy : x > 0\}$ under $T$?

2. Show that $u(x, y) = e^{x^2-y^2} \cos 2xy$ is harmonic and find its harmonic conjugate.

3. Simplify (without a lot of work)

$$
\frac{1}{2\pi} \int_0^{2\pi} e^{\cos^2 \theta - \sin^2 \theta} \cos (2 \cos \theta \sin \theta) \frac{\frac{3}{4}}{\frac{3}{4} - \cos \theta} d\theta.
$$

4. It can be shown that the Taylor series expansion for $\tan z$ is

$$
\tan z = z + \frac{z^3}{3} + \frac{2z^5}{15} + \cdots, \quad |z| < \frac{\pi}{2}.
$$

Let $C$ be the unit circle, traversed once counterclockwise. Evaluate

$$
\int_C \frac{\tan z}{z^2} dz; \quad \int_C \frac{\tan z}{z^3} dz; \quad \int_C \frac{\tan z}{(z - \frac{\pi}{4})^2} dz.
$$

5. Show that $u(z) = 2 \arg(1 + z)$ gives the solution to the Dirichlet problem

$$
\Delta u = 0, \quad |z| < 1
$$

$$
u(e^{it}) = t, \quad -\pi < t < \pi
$$

(Hint: $e^{it} + 1 = e^{it/2}(e^{it/2} + e^{-it/2})$)

6. BONUS: Using $z_0 = iy$ in the previous problem, show that

$$
\int_{-\pi}^{\pi} \frac{tdt}{1 - 2y \sin t + y^2} = \frac{4\pi \tan^{-1} y}{1 - y^2}, \quad |y| < 1,
$$

and/or (for less credit) show me anything else you learned in the course about complex analysis.