1. Recall that a regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

2. For positive integers \( n \), let the numbers \( c(n) \) be determined by the rules \( c(1) = 1 \), \( c(2n) = c(n) \) and \( c(2n + 1) = (-1)^n c(n) \). Find the value of

\[
\sum_{n=1}^{2013} c(n)c(n + 2).
\]

3. Given a positive integer \( n \), what is the largest \( k \) such that the numbers 1, 2, \ldots, \( n \) can be put into \( k \) boxes so that the sum of the numbers in each box is the same? [When \( n = 8 \), the example \{1, 2, 3, 6\}, \{4, 8\}, \{5, 7\} shows that the largest \( k \) is at least 3.]

4. Is there an infinite sequence of real numbers \( a_1, a_2, a_3, \ldots \) such that

\[
a_1^m + a_2^m + a_3^m + \cdots = m
\]

for every positive integer \( m \)?