1. Let $A$ be a solid $a \times b \times c$ rectangular brick in three dimensions, where $a, b, c > 0$. Let $B$ be the set of all points that are a distance at most one from some point of $A$, (in particular $B$ contains $A$). Express the volume of $B$ as a polynomial in $a, b,$ and $c$.

2. How many primes among the positive integers, written as usual in base 10, are such that their digits are alternating 1’s and 0’s, beginning and ending in 1?

3. Determine, with proof, the number of ordered triples $(A_1, A_2, A_3)$ of sets which have the property that (i) $A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and (ii) $A_1 \cap A_2 \cap A_3 = \emptyset$, where $\emptyset$ denotes the empty set. Express the answer in the form $2^a 3^b 5^c 7^d$, where $a, b, c,$ and $d$ are nonnegative integers.

4. What is the units (i.e. rightmost) digit of
$$\left\lfloor \frac{10^{20000}}{10^{100} + 3} \right\rfloor$$?

5. Show that for every positive integer $n$,
$$\left(\frac{2n - 1}{e}\right)^{\frac{2n-1}{2}} < 1 \cdot 3 \cdot 5 \cdots (2n - 1) < \left(\frac{2n + 1}{e}\right)^{\frac{2n+1}{2}}.$$

6. Consider the power series expansion
$$\frac{1}{1 - 2x - x^2} = \sum_{n=0}^{\infty} a_n x^n.$$
Prove that, for each integer $n \geq 0$, there is an integer $m$ such that
$$a_n^2 + a_{n+1}^2 = a_m.$$