Homework 2

1) Let \( f(x) = -x^3 - \cos(x) \) and \( p_0 = 1 \). Use two iterations of Newton’s method to approximate the root of \( f(x) \).

2) The derivative of a function can be approximated by

\[
\frac{d}{dx} f(x) \approx \frac{-3f(x) + 4f(x + h) - f(x + 2h)}{2h}
\]

Show that the error of this approximation is \( O(h^2) \).

3) Use the approximation for \( f''(x) \) given in class to approximate the second derivative of

\[
f(x) = \cos(e^{x^2} + x^3)
\]

at \( x = 1 \) using \( h = .1, .05, .025 \). How does the decrease in error compare to the order of accuracy of the method (is the decrease what we should expect)?

4) Use the composite trapezoidal rule and Simpson’s rule with \( n = 4 \) to approximate

\[
\int_0^1 x^2 e^{-x}
\]

What is the error of each method?

5) Determine \( c_0, c_1, \) and \( c_2 \) such that

\[
\int_0^2 p(x)dx = c_0 p(0) + c_1 p(1) + c_2 p(2)
\]

for all polynomials \( p(x) \) of degree 2 or less.